

Studies in Global Warming and Cooling

by

Nathaniel Tarshish

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Professor David Romps, Chair

Professor Inez Fung

Professor John Chiang

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Abstract

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Which human activities have caused global warming? Using a climate model driven by historical emissions, we demonstrate that global warming is not primarily due to fossil fuels. Instead, agriculture and land use have likely driven historical warming. In fact, fossil fuels probably had a cooling effect throughout most of the 20th century due to their emissions of sulfur aerosols, which increase Earth's reflectivity. This aerosol cooling likely offset the warming induced by greenhouse gases from fossil fuels. However, with sulfur emissions decreasing in recent decades and greenhouse gases continuing to accumulate in the atmosphere due to fossil fuel combustion, the warming attributable to fossil fuels has rapidly risen and is expected to surpass that from other human activities this decade.

Beyond this decade, what long-term temperature is locked in by past emissions? We explore the extent of committed warming by probing the temperature response following the cessation of all current emissions. Using a theoretical model constrained by the best available evidence, we find that ceasing emissions leads to a multi-century temperature decrease of approximately -0.4°C . Thus, future temperature increases beyond present-day are due only to future emissions.

Lastly, we shift our focus to investigating the potential for catastrophic cooling following a limited-scale nuclear exchange. This "nuclear winter" scenario results from a decrease in sunlight caused by soot propelled into the stratosphere by plumes above burning cities. Using a combination of idealized theory and numerical simulations, we demonstrate that these plumes must rely on the latent heat of condensing water to reach climate-altering heights. Consequently, firestorms in geographical regions prone to deep convection pose the highest risk of initiating a nuclear winter.

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Chapter 1

Introduction

Energy flows to Earth via sunlight and streams back to space as both reflected and infrared light. Before the Industrial Revolution, the incoming and outgoing flows of energy were approximately equal, meaning that the planet did not accumulate net energy. Today, however, the planetary energy budget is slightly out of balance. Human activity has reduced the flow of energy back to space by 0.3%. Although this might seem minor, the imbalance results in Earth accumulating an enormous amount of energy *each year*— 10^{22} Joules—equivalent to all of the energy consumed by human activities throughout the entire 20th century.

Which human activities have driven this imbalance? The prevailing wisdom points to fossil fuels as the primary culprit. Powering the Industrial Revolution increased the amount of heat-trapping gases, particularly CO_2 , in the atmosphere, diminishing Earth's ability to cool to space and resulting in higher temperatures. While this narrative holds some truth, we argue here that it is only half the story.

The wrinkle is that fossil fuels, particularly coal and heavy oil, are rich in sulfur. When burned, this sulfur, through a series of chemical reactions in the atmosphere, produces tiny particles known as sulfate aerosols. These aerosols reflect sunlight in two ways: directly, and indirectly, by serving as condensation sites in clouds, thickening them and increasing their reflectivity. Cloudy streaks made in this way by heavy-oil-burning ships can even be seen from space. By increasing the light reflected back to space, these aerosols exert a cooling influence on the climate.

Yet this sulfur, once dissolved in water droplets in clouds, also increase the droplets' acidity, sometimes yielding "acid rain". Because of the negative impacts of acid rain, the environmental movement in the 1980s in the US, and more recently in China and India, has successfully advocated for scrubbing the sulfur out of power-plant emissions. Global sulfur emissions are now half of what they were at their peak in the 1980s. While this is beneficial for preventing acid rain, it poses a challenge for climate control because less sunlight is being reflected by sulfate aerosols.

At the same time as the sulfur cap-and-trade program was being implemented in the US, scientists started to investigate whether the cooling effect of sulfate aerosols was strong

enough to globally compete with the warming caused by greenhouse gases. A conclusive assessment has proven challenging due to the unresolved complexities of cloud physics and uncertain characteristics of aerosols. As a result, aerosols continue to be the principal source of uncertainty in many global climate questions today. Due to these lasting uncertainties and the particles' brief atmospheric lifetime (just a few days), aerosols have frequently been relegated to a secondary role in climate discussions. But did aerosols actually play only a secondary role in historical global warming due to fossil fuels?

Best estimates suggest that these aerosols play an important, but secondary, role at present, masking roughly 50% of the current heating caused by CO_2 . At peak, sulfur emissions were about double what they are now, making this aerosol masking effect even stronger in the past. Back then, CO_2 levels were also lower, and fossil fuels were not as dominant a source of CO_2 emissions; about a third of all CO_2 emissions have historically come from agriculture and land-use changes. So, for much of the 20th century, the CO_2 heating from fossil fuels was lower, yet the aerosol cooling due to these same fuels was higher. Did aerosols more than compensate for greenhouse gases, yielding a net cooling attributable to fossil fuels in the 20th century?

Surprisingly, this question has garnered little attention and is not answered in the state-of-the-science reports. These reports provide the necessary puzzle pieces, but do not put them together to assess the net heating due specifically to fossil fuels, today or in the past. The standard practice is to take the total heating today and divide it into individual contributions due to specific greenhouse gases and aerosols. The concentration of each of these gases is due in part to non-fossil-fuel activities. Elsewhere in these reports are assessments of how much of each gas is emitted by different activities. Here we combine this emissions-per-gas-per-activity data with the heating-per-gas data and then aggregate over the sixty-odd gases and dozens of fossil-fuel activities to assess the net heating due to fossil fuels.

This analysis, detailed in the first chapter, challenges the prevailing wisdom that global warming has been primarily due to fossil fuels. Our findings suggest that, up until this decade, fossil fuels were likely not the leading cause of warming. In fact, prior to the 1980s, the net effect of fossil fuels was likely global cooling, as the cooling effects of aerosols more than offset the warming from greenhouse gases. However, as sulfate emissions have decreased and long-lived CO_2 has accumulated in the atmosphere, an increasing proportion of global heating since 1980 can be attributed to fossil fuels. So why did the temperature rise in the 20th century? Warming was primarily due to the carbon dioxide, methane, and nitrous oxide emitted by agriculture and land-use changes, which lacked a sulfuric mask to hide behind.

While fossil fuels account for half of the human-made heating observed today, they are projected to dominate committed heating (the future heating guaranteed by past emissions) for the next century and beyond. This discrepancy arises because heating due to non-fossil fuel activities is predominantly due to greenhouse gases like methane, N_2O , ozone, and F-gases, which all have shorter lifespans than CO_2 . Over time, the committed impact of non-fossil fuel activities wanes as these shorter-lived gases leave the atmosphere. The warming due to fossil fuels, however, sticks around for millennia. Why is the fossil fuel heating so

long-lasting?

While fossil fuels emit dozens of different gases with varying lifetimes, the first chapter establishes a remarkably simple fact: the current *net* heating from fossil fuels is entirely long-lived and due to CO₂. What about the short-lived cooling aerosols associated with fossil fuels? It turns out this cooling is now exactly counterbalanced by the heating from short-lived non-CO₂ greenhouse gases (mainly methane) also emitted during fossil fuel combustion, ensuring that the net impact of fossil fuels is only long-lived and due to CO₂.

Now that we have identified the activities responsible for historical and committed warming, the second chapter explores whether past emissions commit Earth to higher future temperatures than present. Specifically, if we stopped all emissions today, would temperatures continue to rise? Or is any future temperature increase fundamentally tied to future emissions? This question has naturally received considerable attention. The general scientific consensus, which underpins climate policy, holds that temperatures would remain approximately stable if emissions ceased. To stabilize temperatures, therefore, it is necessary to achieve “net-zero” emissions, where any ongoing emissions are offset by active CO₂ removal.

Upon closer examination, however, we argue that the net-zero paradigm fails to fully capture the complexity of the post-emissions response. The prevailing hypothesis that the temperature remains constant after emissions cease originates from a series of studies in the early 2000s. These studies utilized largely the same climate model to simulate the temperature change after emissions cessation. And, these simulations exhibited little temperature variation after emissions, leading to the foundational premise of carbon budgeting that each emitted molecule of CO₂ permanently ratchets up the temperature.

The argument for the permanence of warming posits a cancellation between two key effects. Without future emissions, carbon dioxide will continue to be absorbed by plants and will diffuse into the ocean, processes that have already removed approximately half of the historical CO₂ emissions. However, the rate of ocean carbon uptake diminishes over time, ultimately leaving about 20% of past emissions stranded in the atmosphere for millennia.

This decline in CO₂ reduces the heating, but temperatures do not necessarily drop in tandem. The other key factor is the deep-ocean heat sink, which moderates surface temperatures by transferring heat to the deep ocean. Over time, however, the deep ocean’s capacity to hide this heat away from the surface diminishes. Whether temperatures rise or fall after emissions cease depends on whether the weakening CO₂ “heater” or the failing deep-ocean “A/C unit” has the stronger effect. In the models used in early studies, these two effects neutralized each other, resulting in a flat temperature response over time.

Since these influential studies, similar experiments have been conducted with many other climate models. These subsequent studies reveal that the post-emissions temperature significantly depends on the model used. Some models predict an eventual 1°C rise in temperature if emissions were to presently cease (indicating that the failing ocean A/C effect dominates), while others show a decrease by the same magnitude (suggesting that the declining heater prevails). Due to this variation among models, there is considerable uncertainty regarding the long-term temperature commitment. This uncertainty compelled the latest U.N. re-

port to express very low confidence that their assessed carbon budgets stabilize the climate beyond the 21st century.

Rather than relying on complex computer models, the second chapter uses an idealized, pencil-and-paper model of energy flows through the Earth system to investigate how temperature evolves after emissions cease. This analysis presents a simple formula that links post-emission temperature changes to several established climate metrics. These metrics measure the degree of carbon disequilibrium in the carbon system and thermal disequilibrium between the upper and lower layers of the ocean. We show that this formula, despite its simplicity, can successfully reproduce the results of published simulations when using those models' specific metrics.

Fortunately, the carbon and thermal disequilibria can be inferred from historical observations, paleoclimate data, and process-based understanding. Evaluating our formula with these input metrics, informed by this more comprehensive body of evidence, shows that it is very likely ($> 90\%$ chance) that temperatures will fall in the centuries after emissions cease. The median estimate is that temperatures will decline by 35% over a multi-century timescale. Our findings clarify that our past emissions have not committed us to a world that is hotter than today. Any additional warming will be due to future emissions. Any rise in temperature above present-day levels will thus result from socioeconomic inertia, not from climate dynamics.

As detailed above, reflection by aerosols plays an important role in shaping Earth's present-day energy imbalance. After emissions cease, however, these aerosols are washed out of the lower atmosphere in just a few days. (Pulling off this aerosol mask spikes the temperature by about $0.2\text{ }^{\circ}\text{C}$, after which the temperature then substantially declines, first due to the disappearance of short-lived greenhouse gases, and then due to the gradual CO_2 uptake by the ocean as described above.) The second part of this thesis (chapters three and four) considers an alternate scenario where aerosols persist in the atmosphere for years rather than days. These aerosols are longer-lived because they reside in the upper atmosphere, above the weather.

High-flying aerosols are implicated in a "nuclear winter": the hypothesis that even a limited, regional nuclear conflict could trigger global starvation. According to this theory, soot from city-scale fires ignited by nuclear weapons would be lofted high into the atmosphere, spread globally, and shade the Earth's surface, resulting in widespread crop failure. Photographs from the nuclear strike on Hiroshima demonstrate the grim reality of such a firestorm plume, although no global climate perturbation was detected following the Hiroshima event, possibly due to the somewhat crude observations available at the time. Since the Hiroshima bombing, nuclear weapons have not only become more numerous but also significantly more powerful. Modern fusion weapons are over 1000 times stronger than the fission bomb dropped on Hiroshima. And, now there are over 10,000 nuclear weapons in the international stockpile.

As a whole, the subject of nuclear winter remains under-investigated and poorly understood. Once a prominent topic of public and scientific concern, interest in nuclear winter

largely waned after the USSR dissolved. It is still uncertain what scale of nuclear conflict would be necessary to trigger a nuclear winter. However, research has provided some insights, indicating that a few million tons of stratospheric soot can cool the Earth by about one degree Celsius over the following year. To put this into perspective, a large-scale city contains about ten million tons of combustible material. Critical questions about the proportion of this material that would turn into soot and the altitude soot would reach in the atmosphere are yet to be answered.

The latter question is the focus of the second half of this thesis: How high does a firestorm plume rise into the atmosphere, and which energy source powers its ascent? The energy from the underlying fire may seem like the obvious answer. Indeed, some recent work has assumed that the plume's rise is exclusively powered by the fire underneath. However, a typical cloud ascends through the atmosphere powered by a different mechanism: the latent heating from the condensation of water vapor. While we commonly experience evaporative cooling — e.g., how our sweat cools our skin — condensational heating is the reverse process, warming the cloud as it rises.

Clouds continue to rise until their temperature equals that of the surrounding environment. Given that the atmosphere gets colder at higher altitudes, why do warm clouds not rise indefinitely? As clouds ascend to lower pressures, they expand and displace the surrounding air. Pushing the environment out of the way extracts energy from the molecules of the rising air, slowing them down and thus lowering their temperature. In dry air, this expansion causes the temperature to drop by about 10°C per kilometer. However, if the air parcel contains ample water vapor, the latent heat released during condensation offsets some of the cooling caused by expansion. As a result, the temperature decline is less severe, approximately -6°C per kilometer the parcel is lifted.

Earth's atmosphere is characterized by a dry 10°C per kilometer temperature profile within the boundary layer, which extends from the surface up to about a kilometer high at the cloud base. Above this, the atmosphere transitions to a moist temperature decline of 6°C per kilometer, typical of cloudy ascent. This profile persists for about the next ten kilometers in a region known as the free troposphere. Above this cloudy layer lies the stratosphere, where the temperature actually increases with height due to the absorption of sunlight by ozone. The stratosphere is the danger zone for nuclear winter, as soot can reside there for months at a time, suspended above weather systems, evading washout by rain, and potentially spreading globally.

How hot must a firestorm plume be to rise all the way to the upper atmosphere without the aid of latent heating? Consider this back-of-the-envelope calculation. The temperature of a dry plume decreases at a rate of 10°C per kilometer, while the background environmental temperature falls off at the slower moist rate of 6°C per kilometer. Thus, any initial temperature *anomaly* relative to the environment will diminish by 4°C per kilometer. To ascend above the weather to near 10 km, given this 4°C per kilometer reduction, a dry plume must start with an initial temperature anomaly of approximately 40°C at the top of the boundary layer.

The third chapter investigates whether a firestorm plume can achieve this temperature by solving for the plume's evolution in the boundary layer. While the fire itself exhibits a temperature anomaly in the hundreds of degrees, the plume's temperature rapidly decreases with height, due to the turbulent whorls and vortices within the plume, which fold in cooler air from the surroundings. Numerical experiments presented in this chapter quantify the strength of this turbulence and provide a model for the plume's temperature decline with altitude.

The last chapter uses this model to determine the temperature of a hypothetical plume over a wide range of potential fire magnitudes and sizes. Powerful fires also generate powerful turbulence, which cools the plume and limits the temperature anomaly of the plume to no hotter than 20°C at the top of the boundary layer for most fires considered. This is significantly cooler than the 40°C requirement for dry ascent to the upper atmosphere. Moreover, this chapter details why 40°C is actually an underestimate of the true requirement, due to the effects of turbulent mixing above the boundary layer.

Firestorm plumes must therefore rely on moisture to reach stratospheric heights, casting doubt on the relevance of earlier literature that neglected the role of water. This dependency suggests that the geographical risk of a nuclear strike leading to nuclear winter is not uniform. In arid environments, stratospheric plume rise is less likely, while in humid regions, particularly those known for vigorous convection, there is a substantially higher risk of soot being transported to climate-altering heights.

This work establishes that the rise of stratospheric firestorm plumes is fundamentally a moist process and requires a humid climate. However, many questions remain unanswered. For instance, how much soot washes out in black rains like those observed over Hiroshima? Moreover, what proportion of combustible material is initially converted into soot? And does the soot absorb enough sunlight to self-loft to even higher altitudes in the months following the fire? These questions must be addressed before the science can bring the threat of nuclear winter into sharper focus.

Chapter 2

Fossil fuels now cause half of anthropogenic warming

Using recent emissions data and a climate emulator, the 2020s are identified as the decade during which fossil-fuel combustion likely becomes the dominant driver of global warming. Prior to 2020, other activities – including agricultural, forestry, and other land use – were likely the major contributors to warming. Prior to 1980, the net effect of fossil fuels was likely cooling, as their generation of sulfate aerosols dominated over the cumulative effect of their greenhouse gases. In recent decades, the accumulation of long-lived carbon dioxide in the atmosphere, alongside a decrease in sulfate emissions, has resulted in a substantial increase in the warming attributable to fossil fuels. This has led to fossil fuels and other activities contributing about equally to anthropogenic warming at present. On the other hand, fossil fuels are responsible for the vast majority of committed warming.

2.1 Introduction

It is commonly asserted that fossil fuels are the primary cause of anthropogenic warming, e.g., as stated by the U.S. National Climate Assessment (Jay et al., 2023), EPA (U.S. Environmental Protection Agency, 2024), NASA (NASA, 2024), and United Nations (United Nations, 2024). Fossil fuel is indeed the main source of carbon dioxide and a major source of methane and nitrous oxide (IPCC, 2021). However, fossil fuel is also the dominant source of sulfate aerosols, tiny particles that *cool* by reflecting sunlight, directly and by increasing the number of water droplets in clouds (Szopa et al., 2021).

The net anthropogenic effective radiative forcing (ERF, also referred to here as the “heating”) for 2023 (relative to 1750) is estimated to be 2.72 [90% CI: 1.96 to 3.48] W m^{-2} , which includes a substantial cooling from anthropogenic aerosols (Wigley, 1989; Hansen et al., 1990; Charlson et al., 1992; Mitchell et al., 1995) estimated to be -1.18 [90% CI: -2.10 to -0.49] W m^{-2} (Forster et al., 2024). A considerable portion of the gross heating derives from

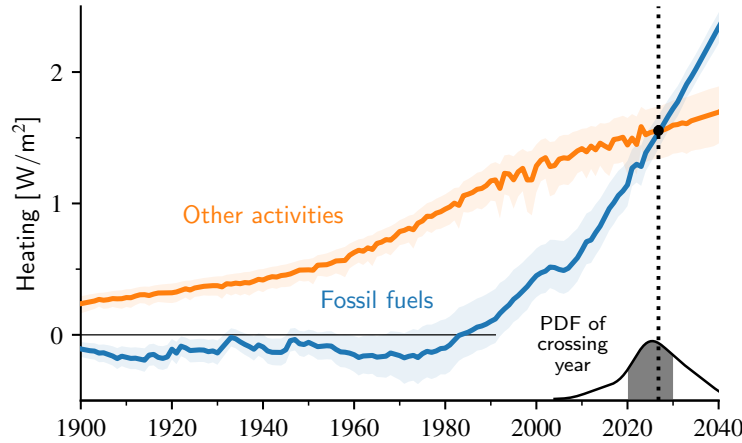


Figure 2.1: Heating (i.e., effective radiative forcing) attributed to burning fossil fuels (blue) versus all other activities (orange) over the historical period. Solid lines are the medians of the climate model ensemble and shading indicates interquartile ranges. The probability distribution function (PDF) of the year in which fossil fuels become the primary source of warming is also shown with the median indicated by a dashed line.

non-fossil-fuel activities, particularly agriculture and land use, while nearly all the aerosol cooling is due to fossil fuels. The fossil-fuel origin of this cooling has raised questions about the extent to which historical temperature increases have been driven by fossil fuels (Shindell and Faluvegi, 2010; Dreyfus et al., 2022), especially given that fossil-fuel CO_2 emissions were lower, and sulfate emissions higher, in the past.

Notably, the Intergovernmental Panel on Climate Change (IPCC) has not assessed whether fossil-fuel combustion is the dominant source of present-day global warming. Instead, the IPCC Sixth Assessment (AR6) report follows the standard practice of attributing heating to the concentrations of individual greenhouse gases and aerosols (IPCC, 2021). Since current concentrations are due to both fossil fuels and other activities, the impact of fossil fuels alone is unclear. This present study fills this gap by providing a probabilistic evaluation, consistent with the state of the science in AR6, of the contribution of fossil fuels to global warming over the industrial era.

2.2 Fossil fuels shift from cooling to warming

The impact of fossil fuel emissions on global warming is analyzed here using a simple climate model ensemble (Smith et al., 2024). Recent IPCC reports prominently feature this same model, the Finite-amplitude Impulse Response (FaIR) emulator (Myhre et al., 2013; Masson-

Delmotte et al., 2018b; Forster et al., 2021a), which was specifically developed to assess global-mean climate variations caused by greenhouse gases and aerosols (Smith et al., 2018; Nicholls et al., 2020; Leach et al., 2021b). Experiments are performed here with an 841-member ensemble (Smith et al., 2024), calibrated to reproduce historical observations and AR6-assessed ranges for key climate metrics, including the uncertainty of the magnitude of aerosol cooling (see Appendix).

Historical emissions data are sourced from the same datasets used in AR6 (van Marle et al., 2017; Hoesly et al., 2018), updated to the present day as per the 2023 Indicators of Global Climate Change (IGCC) report (Gütschow et al., 2016; van der Werf et al., 2017; Friedlingstein et al., 2023a; Hoesly and Smith, 2024; Forster et al., 2024). Future emissions and gaps in historical data, arising from reporting delays, are estimated by linear extrapolation of the past decade’s emissions for each species and activity. When all emission sources are considered, the FaIR ensemble agrees with the IGCC on the central estimates and uncertainties for total and per-species radiative forcings (refer to Appendix Figure 2.7).

Anthropogenic heating is attributed here to the burning of fossil fuels and other activities using two experiments with FaIR: one driven solely by historical fossil-fuel emissions, and another driven by historical emissions from all other activities. Those other activities are agriculture, forestry, other land uses (AFOLU), cement calcination, biomass combustion, certain mineral and chemical processes, waste handling, landfills, and emissions of halogenated gases. For a detailed breakdown of the emissions sources and classification, refer to Appendix. The FaIR ensemble solves for the gas concentrations, forcings, and global temperature anomaly given each experiment’s subset of historical emissions.

Due to nonlinear effects (primarily from atmospheric chemistry), the sum of the radiative heating from these two separate experiments exceeds the heating due to all emissions by 11% in 2024 (see Appendix Figure 2.11 and Table 2.2). Since this nonlinear component of the total heating is generated if and only if both sets of emissions are present, it is attributed here in equal parts to fossil fuels and other activities. The heating timeseries for the two separate experiments are reduced by equal (yet time-varying) amounts so that their sum matches the heating timeseries resulting from when FaIR is driven by all emissions combined (see Appendix).

Figure 2.1 shows the resultant heating timeseries for the fossil-fuel experiment in blue and for other activities in orange. The median estimate of net fossil-fuel radiative forcing is negative, averaging -0.1 W m^{-2} , up until the early 1980s. This suggests that fossil fuels contributed to a global cooling effect during much of the 20th century, partially offsetting warming from other sources. From the 1980s onward, heating from fossil fuels has increased sharply, rising at 0.3 W m^{-2} per decade, and culminating in a total of 1.4 W m^{-2} by 2024. In contrast, the heating from other sources (mainly AFOLU) has been consistently positive throughout the 20th century, with a slower average increase of 0.1 W m^{-2} per decade since 1980.

To understand changes in warming over time, Figure 2.2 categorizes radiative forcings as either long-lived or short-lived. All species, except for CO_2 , N_2O , and halogenated gases that

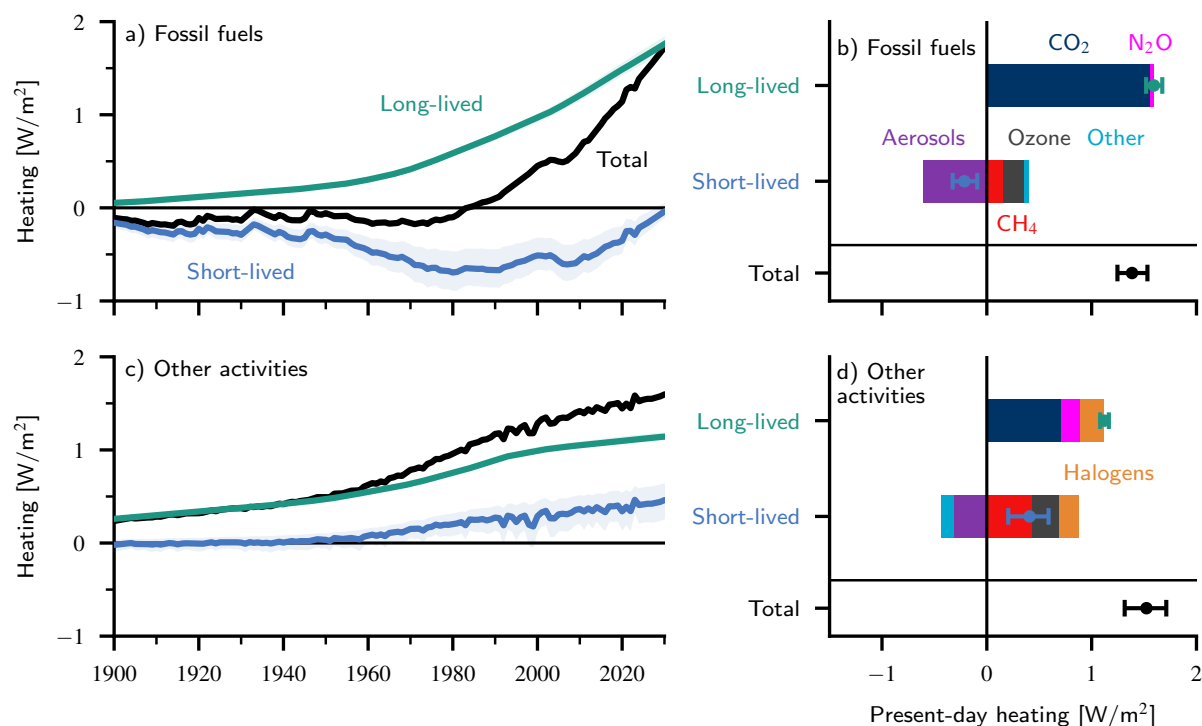


Figure 2.2: a) Heating from long-lived fossil-fuel greenhouse gases (green) has accumulated over time, overcoming cooling due to co-emitted short-lived species (blue) around 1980. b) The long-lived component of fossil fuels is predominantly from CO_2 , while aerosols have dominated the short-lived species. c) Warming from other activities (primarily agriculture and land use) has grown relatively slowly since 1980 and has never been substantially masked by short-lived cooling. d) The short- and long-lived heating due to other activities are both positive and involve a variety of species. Shadings and whiskers indicate interquartile ranges.

have lifetimes of a century or more are considered short-lived. Short-lived gas concentrations are proportional to recent emissions levels, while long-lived gas concentrations are determined by cumulative emissions over the entire industrial period.

At the turn of the 20th century, annual CO_2 emissions from fossil fuels and other activities were the same order of magnitude (Appendix Figure 2.4). However, the heating effect from fossil-fuel CO_2 was minimal because not enough time had passed since the start of industrialization for those emissions to accumulate significantly. In contrast, CO_2 from non-fossil sources had been accumulating from deforestation over the prior centuries, resulting in substantial radiative forcing by 1900.

Sulfate concentrations, however, depend only on recent emissions, meaning that aerosol cooling from fossil fuels had a substantial influence even at the onset of industrialization. Additionally, the fuel mix in the early 20th century had a higher sulfur content than present-day (Hoesly et al., 2018), further enhancing the relative importance of aerosol cooling. As a result, fossil fuels yielded a net cooling over much of the 20th century: short-lived cooling outweighed the gradual build up of long-lived greenhouse gases, primarily CO_2 . This dynamic shifted around 1980 when the fossil-fuel CO_2 had finally accumulated to sufficient levels to overcome aerosol cooling and produce net heating.

The sharp increase in fossil-fuel-related heating since then can be attributed to a significant rise in long-lived greenhouse gas emissions, and also a reduction in short-lived cooling effects, primarily from a decrease in sulfate aerosols (Appendix Figure 2.9). Stricter emissions standards that led to the adoption of flue-gas desulfurization at coal-fired power plants and limits on fuel sulfur content are responsible for this decline in aerosols (Liu et al., 2018).

Currently, the net effect of short-lived forcings from fossil fuels is negligible, as the cooling from aerosols is largely balanced by heating from ozone and methane (Figure 2.2b). Therefore, existing fossil-fuel infrastructure is adding additional heating that can be thought of, to good approximation, as entirely long-lived and due to CO_2 . In contrast, much of the heating due to other activities is short-lived. Furthermore, compared to fossil fuels, the growth in the long-lived component of heating from other activities has been relatively modest over recent decades.

The closest comparison to our findings is work of Dreyfus et al. (2022), which approximated fossil fuel's share of each species' current radiative forcing by the fraction of fossil fuel's cumulative emissions of that species since the preindustrial. That method does not account for the natural sinks of those species, the timescales of those sinks, or the time series of emissions, all of which are accounted for here. Whereas the approximate method estimated that only 34% of anthropogenic heating in 2019 was due to fossil fuels (Dreyfus et al., 2022), the median fraction for 2019 is found here to be 42%, approaching the parity of 50% that is most likely to occur in the current decade.

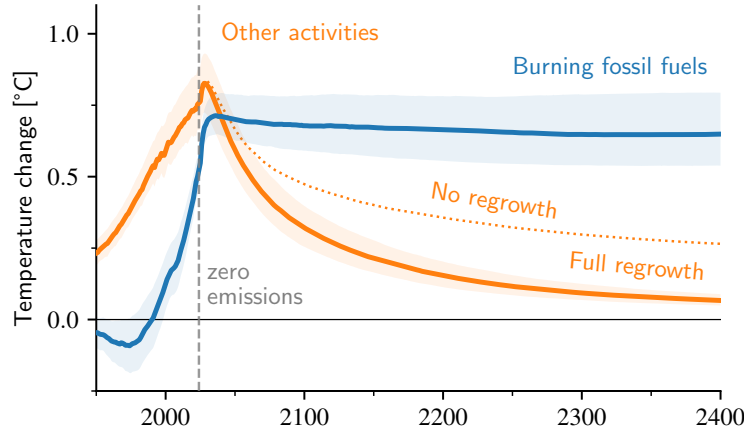


Figure 2.3: The temperature response to presently ceasing emissions in scenarios driven solely by burning fossil fuels (blue) and all other activities (orange), respectively. In the latter case, the fate of agriculture and land-use CO_2 emissions is illustrated by scenarios with no (dashed) and full (solid) regrowth from ecosystem restoration.

2.3 Fossil fuels now at parity with other activities

Figure 2.1 illustrates that the 2020s are the most likely decade for fossil fuels to overtake other activities to become the leading cause of global warming. The probability distribution function (PDF) of crossing times (from the 841 members of the ensemble) peaks this decade with a mode of 2024 and median of 2026. In addition, the ensemble gives a 52% chance that the crossing occurs this decade.

The range of predicted crossing times in the ensemble reflects uncertainties within the climate system, primarily related to the direct and indirect radiative effects of aerosols (see Appendix Figure 2.6 and Table 2.2). Figure 2.1, however, does not account for uncertainties in the emissions data. Emissions uncertainties are expected to be symmetrical (Forster et al., 2024; Friedlingstein et al., 2023a), such that accounting for them would broaden the PDF of crossing times but not shift the peak, meaning that the 2020s would still be the most likely decade for fossil fuels to begin dominating warming.

Finally, we note that reporting practices make it unclear whether certain industrial process and product use (IPPU) emissions originate from fossil fuels (Intergovernmental Panel on Climate Change, 2019). In Figure 1, these ambiguous IPPU emissions are categorized as non-fossil. If these ambiguous IPPU emissions are reclassified as originating from fossil fuels, the median crossing year moves to 2025 (refer to Appendix Figure 2.12). In either case, this decade is the most likely time for the transition to fossil-fuel dominance.

2.4 Committed warming dominated by fossil fuels

While fossil fuels are just now rivalling other activities in terms of instantaneous heating, fossil fuels began dominating the heating from long-lived species in the early 2000s. This means that the commitment to future temperature anomalies were already dominated by fossil fuels around the turn of the century. Today, in 2024, the long-lived heating from fossil fuels is around 40% larger than that of other activities.

The previous sections separated species into short-lived and long-lived using a timescale of a century, but it is possible to consider the commitment to temperature anomalies at much longer timescales. In the centuries beyond 2100, some “long-lived” species like N_2O and many types of halogens are effectively removed, leaving CO_2 responsible for the vast majority of the heating. The cumulative emissions of CO_2 due to fossil fuels surpassed other activities in the mid-1980s, so the multi-century heating commitment has been dominated by fossil fuels for several decades.

Furthermore, the CO_2 emissions from fossil fuels are fundamentally different from the CO_2 emissions from other activities, which come mainly from AFOLU. If future food production shifts away from land-intensive practices, past land-use CO_2 emissions would largely be removed from the atmosphere due to regrowth on a centennial timescale (Nabuurs et al., 2022). On the other hand, carbon dioxide from fossil fuels is primarily taken up by the ocean, on a set of timescales ranging from centuries to many millennia (Archer et al., 1997).

Figure 2.3 explores the temperature commitments from fossil fuels and other activities. In these experiments, emissions are ceased in 2024. Although the present-day total radiative forcings in each experiment are similar (as shown in Figure 2.1), the temperature increase in the fossil-fuel case lags by 0.2 K due to the ocean’s thermal inertia.

The temperature anomaly due to fossil fuels remains essentially constant over the coming centuries. Radiative forcing, on the other hand, is roughly halved in this scenario over the following centuries due to CO_2 uptake by natural carbon sinks (Appendix Figure 2.10). However, this decline in heating is balanced by a weakening of the deep-ocean heat sink, which is substantially moderating surface temperatures at present. The compensation between the declining source and sink of heat results in a time-invariant temperature commitment from burning fossil fuels (Matthews and Caldeira, 2008; Solomon et al., 2009).

In contrast, the temperature change due to other activities declines steeply after emissions cessation, particularly under the assumptions of strong regrowth (see Appendix). Large declines in radiative forcing due to the shorter-lived nature of these forcings dominate over the waning oceanic heat sink, leading to a global-average temperature that approaches pre-industrial levels on this multi-century timescale. Thus, the commitment to temperature anomalies on multi-centennial timescales (Palazzo Corner et al., 2023) is overwhelmingly due to fossil fuels.

2.5 Discussion

These findings underscore the urgency of reducing the burning of fossil fuels relative to other mitigation activities. Fossil-fuel heating has grown rapidly over the last few decades, yielding a temperature increase that is effectively permanent on civilizational timescales absent negative-emissions technologies. At the same time, the analysis here challenges the prevailing belief that fossil-fuel combustion was the main driver of 20th-century warming (Jay et al., 2023; NASA, 2024; U.S. Environmental Protection Agency, 2024; United Nations, 2024). Until this decade, it is likely that fossil fuels were *not* the primary cause of global warming. Looking forward, however, as the reliance on fossil fuels continues (International Energy Agency, 2023) and tighter emissions controls on sulfate aerosols are implemented (Riahi et al., 2017), the heating from fossil fuels will increasingly dominate future warming.

Appendix

Historical emissions

Emissions of greenhouse gases and aerosols from 1750 to the present are sourced from the same datasets used by the IGCC (Forster et al., 2024), which applies the AR6 methodology to the most recent data. Fossil-fuel emissions include all sources except those from agriculture, forestry, and other land uses (AFOLU); cement calcination and other material processes; specific industrial chemical and mineral processes; biomass fires and waste-to-energy; waste management; landfills; and halogenated gases. Guided by the IPCC’s common reporting framework (Intergovernmental Panel on Climate Change, 2019), the fossil-fuel category defined here covers all energy sector emissions except for biomass combustion and includes a subset of the emissions due to industrial processes (Sector 2), waste (Sector 6), and other activities (Sector 7).

All CO₂ emissions data are taken from the 2023 Global Carbon Project (GCB) (Friedlingstein et al., 2023b). Fossil-fuel CO₂ emissions are taken from the fossil-only category of the GCB dataset, while non-fossil-fuel CO₂ emissions are sourced from GCB’s land-use-change and calcination datasets.

The Community Emissions Database (CEDS) (Hoesly et al., 2018; Hoesly and Smith, 2024) is used to source emissions from 1750 to 2019 of black carbon (BC), organic carbon (OC), NH₃, NO_x, SO₂, and volatile organic compounds (VOC). From 2019 to 2022, the CEDS data are projected forward using emission factors based on mobility data from the two-year blip scenario of COVIDMIP (Lamboll et al., 2021) following the methodology of Smith et al. (2024). CEDS covers all activities except for biomass fires. The CEDS dataset categorizes fossil-fuel emissions by sector, detailed in Table 2.1. Biomass combustion emissions are excluded from the energy sector using data from the alternative “by fuel” CEDS dataset. The role of fossil fuels in emissions from certain industrial sectors remains ambiguous due to

the diverse chemical processes aggregated within these sectors. While some of these processes undoubtedly use fossil fuels as feedstocks, precise attribution is not available in the CEDS dataset. In the primary results and figures, these ambiguous sectors are grouped with non-fossil activities. However, attributing these ambiguous emissions to fossil fuels instead does not affect the conclusions (see Appendix Figure 2.12).

The primary source for CH_4 and N_2O emissions is PRIMAP-Hist (i.e., the “third-party” version) (Gütschow et al., 2016; Gütschow and Pflüger, 2023), supplemented by CEDS data where necessary. PRIMAP data are grouped at the IPCC sectoral level, necessitating a further breakdown for the waste and industrial sector emissions, which include fossil, biogenic, and ambiguous sources. CEDS, which provides data at the finer subsector level, is used to estimate the fractional contribution of the above sources to the PRIMAP sector totals. The CEDS record, however, only goes back to 1970. To disaggregate the PRIMAP waste emissions before 1970, the fraction of PRIMAP sectoral emissions due to waste combustion is assumed to be constant prior to 1970. A similar approach is used to categorize non-fossil and ambiguous activities within the Industrial Process and Product Use (IPPU) sector based on CEDS subsector data, with these respective emissions fractions assumed constant before 1970.

Emissions from biomass burning of all species other than CO_2 (i.e., BC, CH_4 , CO, NH_3 , NO_x , OC, and SO_2) are sourced from the Global Fire Emissions Dataset (van der Werf et al., 2017) and from BB4CMIP (van Marle et al., 2017). Halogenated gas emissions (F-gases) come from Smith et al. (2024) and are inverted from observed concentrations using IPCC assessed lifetimes, except for SF_6 and NF_3 , which are available in the PRIMAP-Hist dataset. Future emissions and historical data gaps due to reporting delays are addressed by linear extrapolation based on per-species decadal emissions trends.

Climate emulator

Reduced-complexity models are well-suited to attributing radiative forcing to specific activities because they can be used in large ensembles that span key uncertainties and can be calibrated against observed data and other climate metrics (Nicholls et al., 2020; Smith et al., 2024). Specifically, the AR6 WGIII report included a detailed calibration of several such models, including the Finite-amplitude Impulse Response (FaIR) model, used in this study (IPCC, 2022). Simulations are performed here with the latest version (2.1.0) (Leach et al., 2021b), which has been calibrated to observations updated through 2022 in Smith et al. (2024) to provide an 841-member AR6-consistent ensemble (v1.4.1). The complete calibration process is documented in (Smith et al., 2024) and summarized here for clarity.

FaIRv2.1 simulates global-average quantities with a three-box ocean model and a basic carbon-cycle parameterization with four uptake timescales (Millar et al., 2017; Smith et al., 2018; Leach et al., 2021b). Those four atmospheric lifetimes of CO_2 vary based on atmospheric levels, global-mean temperature, and the cumulative uptake by land and oceans. Methane’s lifetime depends on the global-mean temperature and the presence of

other chemically-active species, including methane itself, as illustrated in Appendix Figure 2.8. Aside from CO_2 and CH_4 , all other greenhouse gases (GHGs) decay at a constant rate over a predefined timescale. The model also simulates interactive ozone levels – combined tropospheric and stratospheric – and parameterizes aerosol direct and indirect effects using logarithmic functional fits (Eqn. 8 in (Smith et al., 2024)). The sensitivity of this aerosol forcing to various species is illustrated in Appendix Figure 2.9. Changes in albedo from land-use are assumed to scale linearly with cumulative AFOLU CO_2 emissions (see Smith et al. (2018) for details).

The 841-member ensemble used in this study is a subset an initial 1.6 million member ensemble sampled from AR6 probability distributions (Smith et al., 2024), chosen as follows. Each of the 1.6-million model instances has 45 free parameters. Parameters related to energy balance, the carbon cycle, ozone, and aerosol-cloud interactions are sampled from kernel-density estimates of CMIP6 results (Skeie et al., 2020; Thornhill et al., 2021a,c; Arora et al., 2020). The other forcing parameters are sampled from IPCC AR6 ranges (Lee et al., 2021a). Models that fail to replicate historical global-mean surface temperatures (GMST) within a root-mean-square error (RMSE) of less than 0.2 K are excluded, eliminating 86% of the original ensemble. From the remaining models, 841 are selected that together optimally reproduce the AR6 distributions for equilibrium climate sensitivity (ECS), transient climate response (TCR), and aerosol forcing, as well as historical observations of GMST, ocean heat content, and CO_2 levels (Smith et al., 2024).

Simulations

Changes in temperature and forcing are assessed relative to their levels in 1750. The “fossil fuel” experiments are driven by the changes in emissions since preindustrial (1750) due to solely fossil-fuel activity. In these fossil-fuel experiments, emissions from non-fossil-fuel activities are held constant at their preindustrial baselines. Their inclusion is necessary to apply the aerosol forcing formulae in FaIR (Smith et al., 2024), which are tuned relative to the total pre-industrial emissions. The “other activities” experiments are likewise driven by the historical changes in the emission since preindustrial due solely to non-fossil-fuel activities, holding fossil-fuel emissions constant at their 1750 levels.

The first set of experiments (shown in Figures 2.1 and 2.2) follow a linear projection of emissions to 2040 based on the historical growth over the last decade of each respective activity’s emissions. The probability density function (PDF) shown in Figure 2.1 for the year when fossil-fuel activities overtake other activities as the dominant cause of global warming is computed by fitting a Gaussian kernel density (KDE) estimate with bandwidth determined by Silverman’s rule (Silverman, 1986) to the ensemble’s distribution of crossing years. In Appendix Figure 2.12, the cumulative distribution function is also shown.

In the second set of experiments (shown in Figure 2.3), historical emissions abruptly end in 2024, probing the zero-emissions commitment (Palazzo Corner et al., 2023) out to 2500. We explore contrasting futures of land activity after emissions with the “no regrowth” and

“full regrowth” scenarios. In the no-regrowth scenario, current levels of land activity are maintained, with no additional carbon uptake beyond the natural sinks accounted for by the FaIR model. In the full-regrowth scenario, land-use stops, allowing ecosystems to recover. This scenario adds an additional carbon sink to account for regrowth on abandoned land, removing AFOLU emissions from the atmosphere with an e -folding timescale of one century. This simple treatment of carbon uptake by soil and biomass from post-agricultural succession is relatively conservative. Over half of historical land-use emissions originate from tropical regions (Houghton and Castanho, 2023), where ecosystems typically regrow over several decades, according to biomass versus age studies of abandoned land (Marìn-Spiotta et al., 2008). However, we conservatively use the regrowth timescale of boreal forests, which is closer to a century, as found in studies of regrowth after disturbances (Tompalski et al., 2024).

Nonlinearity

Summing the total radiative forcings from the separate fossil-fuel and other-activities experiments results in a value that is 11% greater than when FaIR is driven by all emissions combined (see Table 2.2). The time series of this nonlinearity is displayed in Appendix Figure 2.11, and computed by differencing the heating timeseries from FaIR driven by all emissions with the sum of the heating timeseries of the two separate experiments. The nonlinearity in forcing is most pronounced for methane, both directly and indirectly via its effect on ozone. Changes in methane chemistry are largely driven by NO_x (Thornhill et al., 2021a). As detailed in Appendix Figure 2.8, experiments that include fossil-fuel NO_x exhibit a halving of methane’s lifetime over the industrial era. Here we attribute the nonlinear component of the methane heating equally to the two experiments. Thus, half of the (negative) heating due to these nonlinear effects (shown in Appendix Figure 2.11) is added to the heating timeseries of the fossil-fuel and other-activities experiments. This process is repeated for each species such that the total forcing resulting from the sum of the separate experiments is scaled to match that of FaIR when driven by all emissions combined. While there are nonlinearities in the relationship between emissions and forcing, the energy-balance model in FaIR that computes the temperature from the forcings is linear.

Table 2.1: Assignment of CEDS Hoesly et al. (2018) subsectors to fossil fuels and other activities. In the main figures, the ambiguous column is grouped with the other activities. Supplementary Figure 2.12 demonstrates the impact of alternatively associating these ambiguous emissions with fossil fuels.

Fossil Fuels	Other	Ambiguous
1A1a_Electricity-autoproducer	2A1_Cement-production	2Ax_Other-minerals
1A1a_Electricity-public	2A2_Lime-production	2B2_Chemicals-Nitric-acid
1A1a_Heat-production	3B_Manure-management	2B3_Chemicals-Adipic-acid
1A1bc_Other-transformation	3D_Rice-Cultivation	2B_Chemical-industry
1A2a_Ind-Comb-Iron-steel	3D_Soil-emissions	2D_Chemical-products-manufacture-processing
1A2b_Ind-Comb-Non-ferrous-metals	3E_Enteric-fermentation	2D_Degreasing-Cleaning
1A2c_Ind-Comb-Chemicals	3F_Agricultural-residue-burning-on-fields	2D_Other-product-use
1A2d_Ind-Comb-Pulp-paper	3I_Agriculture-other	2D_Paint-application
1A2e_Ind-Comb-Food-tobacco	5A_Solid-waste-disposal	2H_Pulp-and-paper-food-beverage-wood
1A2f_Ind-Comb-Non-metalic-minerals	5D_Wastewater-handling	2L_Other-process-emissions
1A2g_Ind-Comb-Construction	5E_Other-waste-handling	
1A2g_Ind-Comb-machinery		
1A2g_Ind-Comb-mining-quarrying		
1A2g_Ind-Comb-other		
1A2g_Ind-Comb-textile-leatherp		
1A2g_Ind-Comb-transpequip		

Table 2.1 – continued from previous page

Fossil Fuels	Other	Ambiguous
1A2g_Ind-Comb-wood- products		
1A3ai_International- aviation		
1A3aai_Domestic- aviation		
1A3b_Road		
1A3c_Rail		
1A3di_International- shipping		
1A3di_Oil_Tanker_Loading		
1A3di_Oil_tanker_loading		
1A3dii_Domestic- navigation		
1A3eii_Other-transp		
1A4a_Commercial- institutional		
1A4b_Residential		
1A4c_Agriculture- forestry-fishing		
1A5_Other-unspecified		
1B1_Fugitive-solid-fuels		
1B2_Fugitive-petr		
1B2_Fugitive-petr-and- gas		
1B2b_Fugitive-NG-distr		
1B2b_Fugitive-NG-prod		
1B2d_Fugitive-other- energy		
2C_Metal-production		
5C_Waste-combustion		
6A_Other-in-total		
6B_Other-not-in-total		
7A_Fossil-fuel-fires		
7BC_Indirect-N ₂ O-non- agricultural-N		

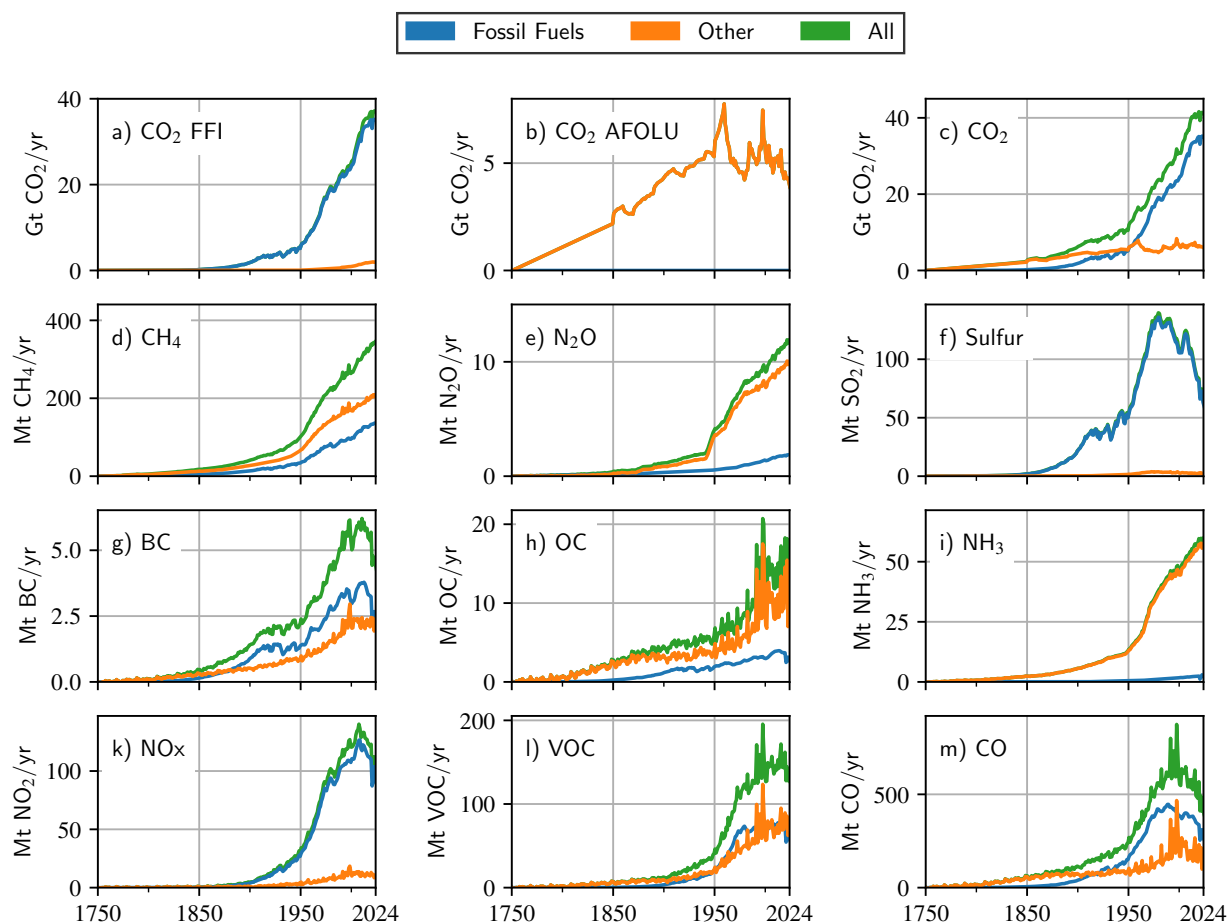


Figure 2.4: The change in emissions from preindustrial due to fossil-fuel activities (blue), other activities (orange), and all activities combined (green) for various species. The changes due to all activities are identical to the results of Smith et al. (2024) except for N₂O, which slightly differs to the inclusion of N₂O land-use, land-use change, and forestry (LULUCF) emissions in this study.

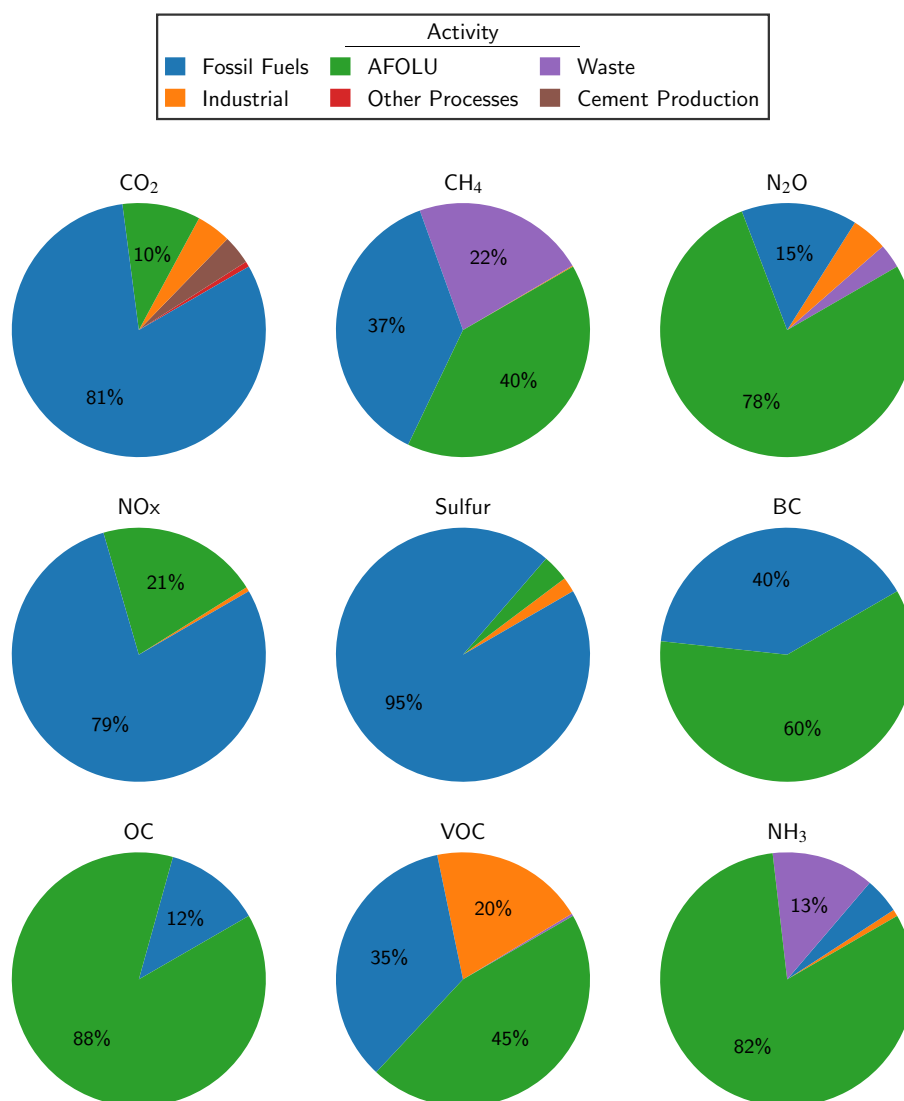


Figure 2.5: The fractional contribution of different activities to the emissions of key species. Here, the blue “Fossil Fuels” activity encompasses all fossil-fuel emissions. Note that AFOLU here includes emissions from open biomass burning (GFED) and biomass combustion for energy purposes (CEDs).

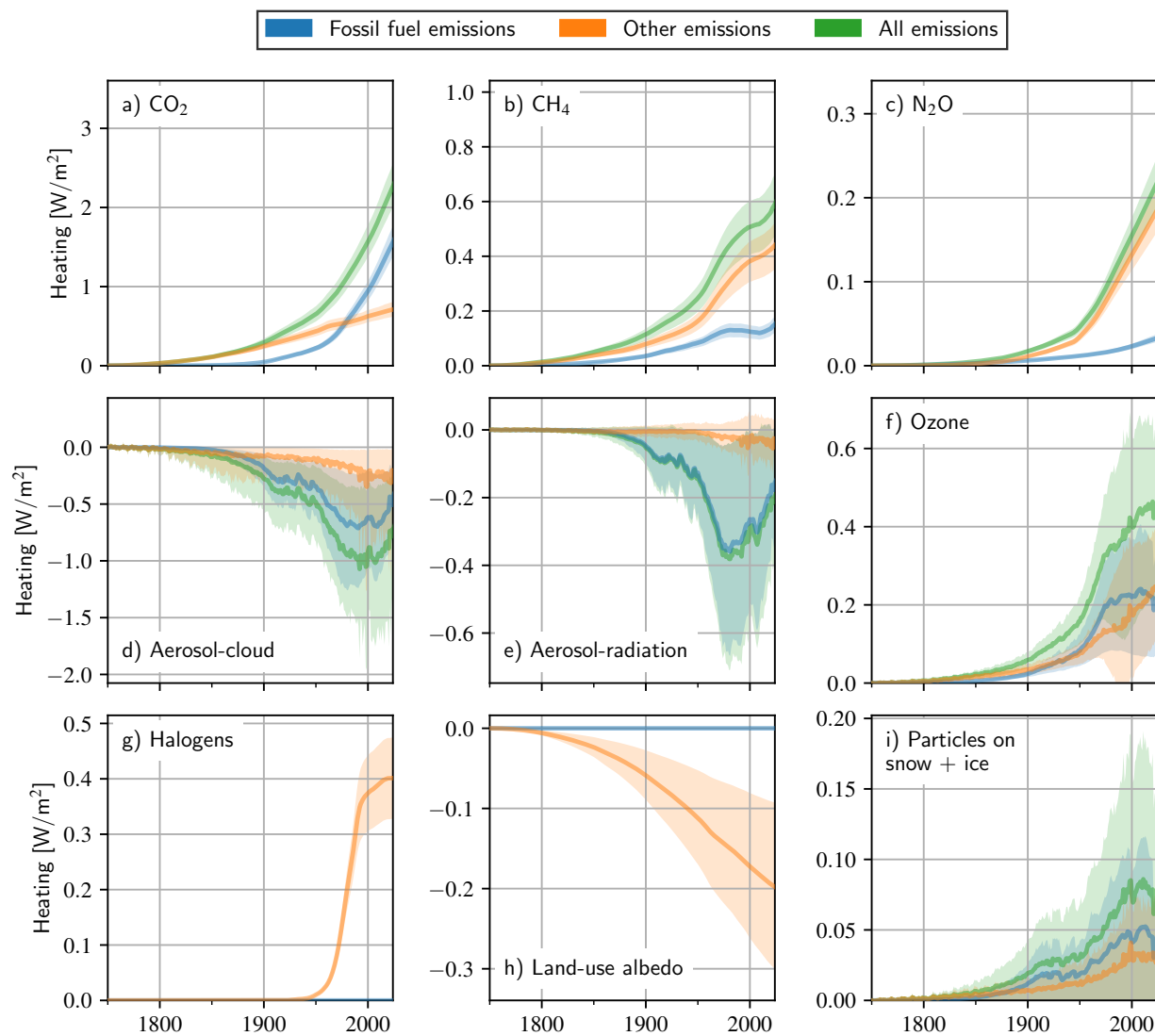


Figure 2.6: Effective radiative forcings computed by the FaIR ensemble given the change in emissions since preindustrial due to all activities (green), solely fossil fuels (blue), and other activities (green). Solid lines are median estimates and shading represents the 5th-95th interquartile range.

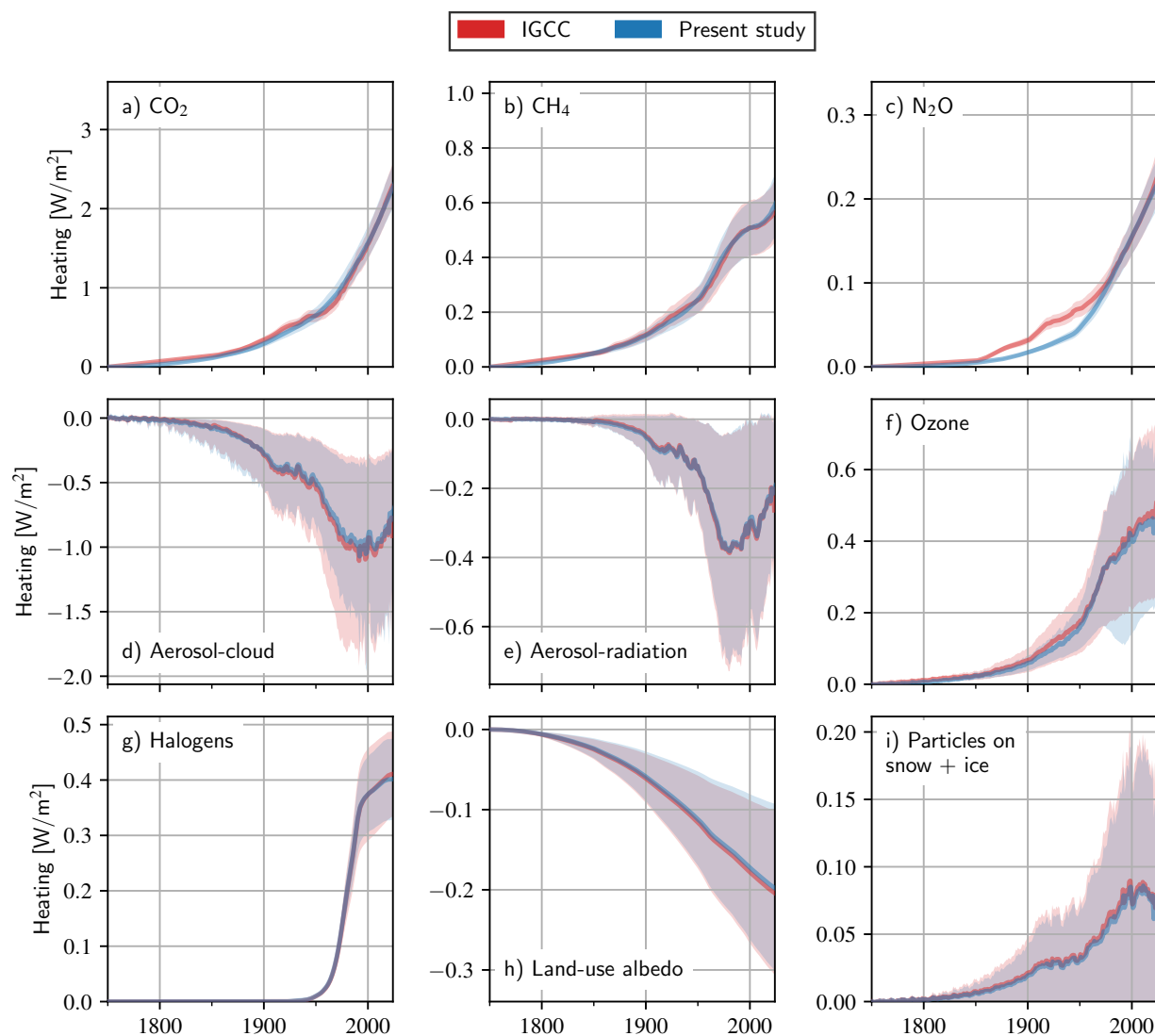


Figure 2.7: Effective radiative forcings computed by the FaIR ensemble given all historical emissions (blue) compared to the IGCC estimates (red) for various species. Solid lines are median estimates and shading represents the 5th-95th interquantile range.

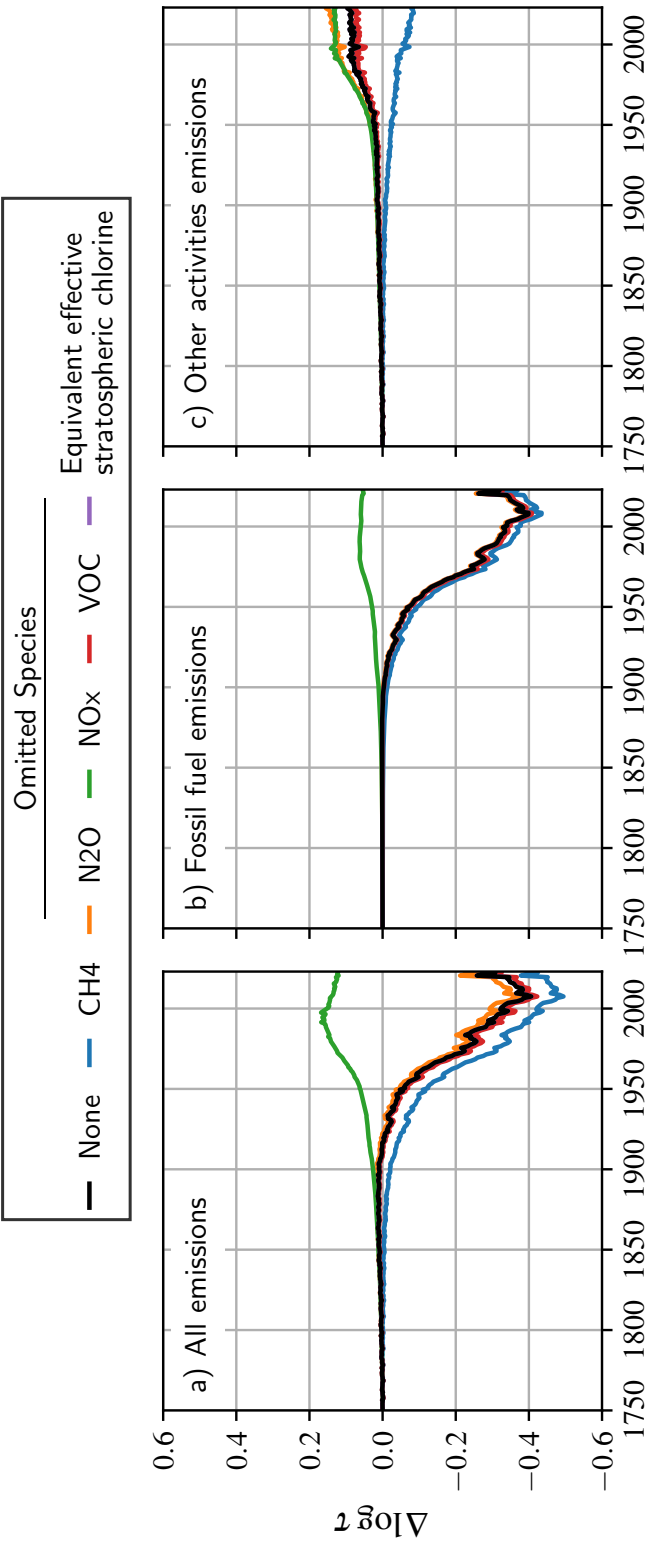


Figure 2.8: The sensitivity of methane’s lifetime (τ) to various species illustrated through a series of perturbation experiments. Each experiment computes the change in the natural logarithm of methane’s lifetime from its preindustrial value by holding the emissions of one species at preindustrial levels while allowing others to follow historical trends. The curves are labeled according to the species whose emissions are held fixed to preindustrial.

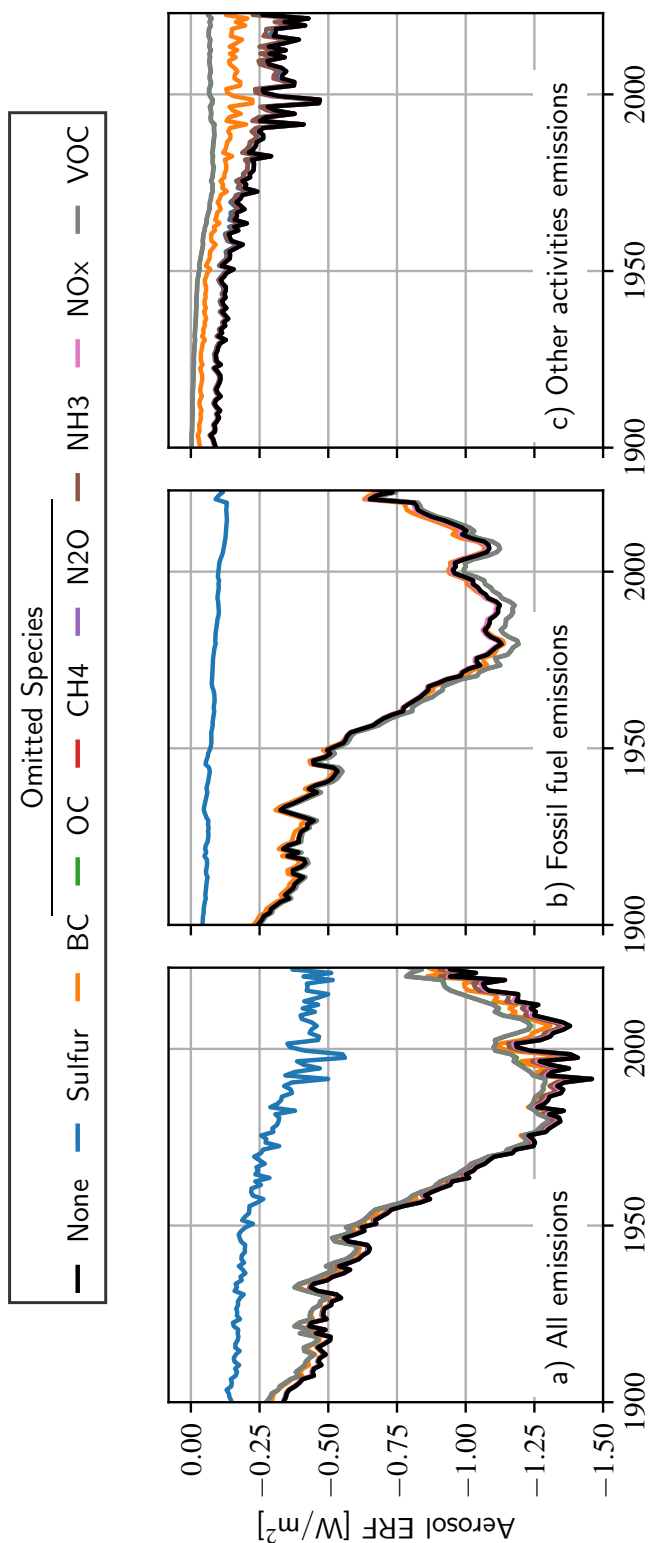


Figure 2.9: Same as Figure 2.8 but for the total effective radiative forcing (ERF) due to aerosols.

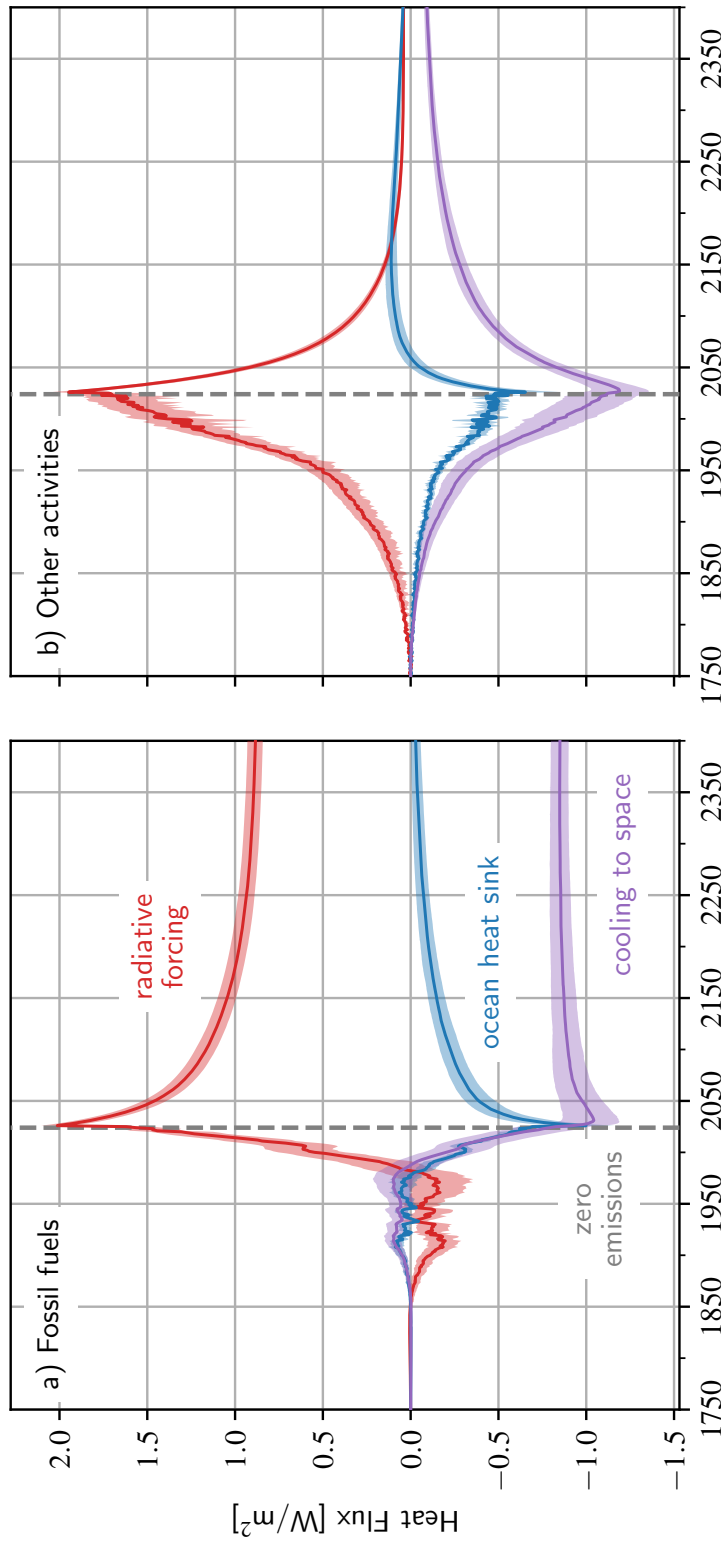


Figure 2.10: a) The decline in radiative forcing (red) by roughly 1 W/m^2 is compensated for by a decrease in cooling to the deep ocean (blue) of equal magnitude after the cessation of fossil-fuel emissions. The temperature, proportional to cooling to space (purple), exhibits a post-emissions plateau due to this cancellation. b) Uptake in AFOLU CO_2 and the disappearance of short-lived forcings yields negligible forcing due to other activities on multi-centennial timescales, yielding a dramatic temperature decline.

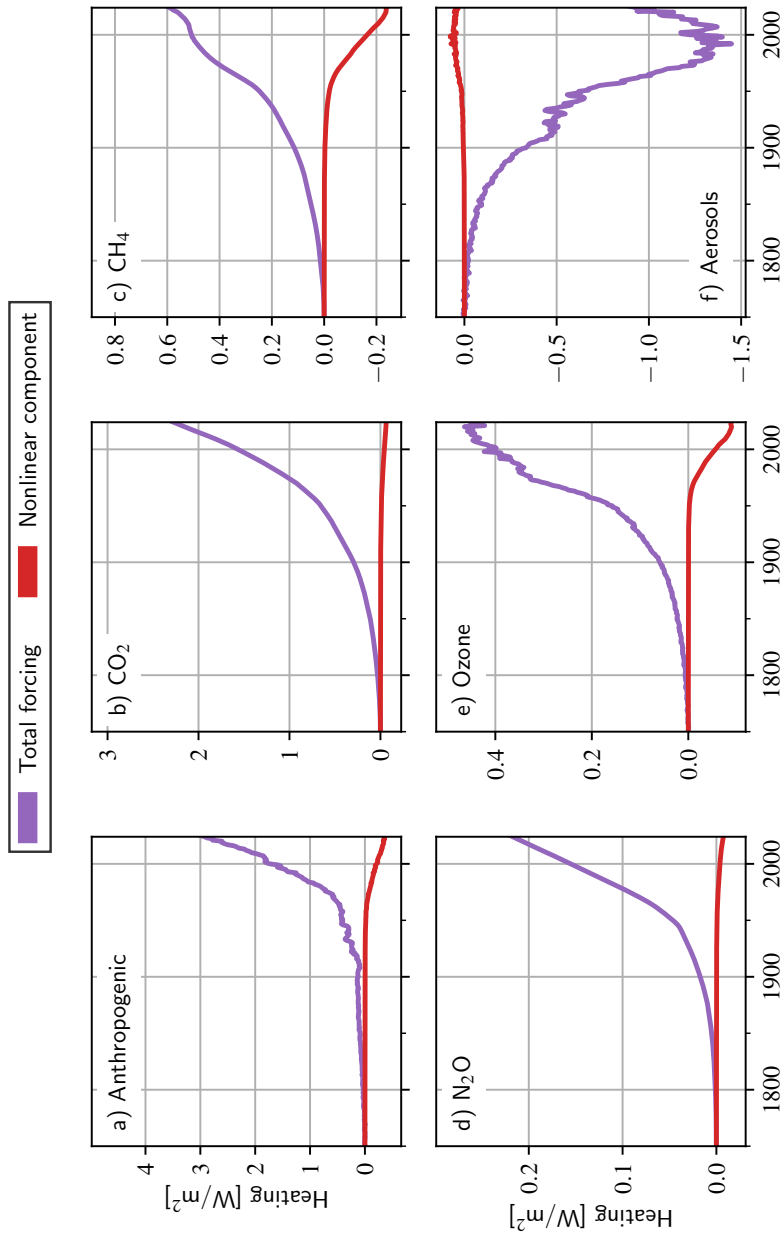


Figure 2.11: The nonlinear component (red) arises from the interaction between fossil fuel emissions and emissions due to other activities. This nonlinearity makes the heating in the all-emissions experiment (purple) less than the sum of the heatings from the fossil-fuel and other-activities experiments. The nonlinear contribution (red) is calculated by subtracting the sum of the heatings from the fossil-fuel and other-activities experiments (not shown) from the total forcing observed when all emissions are considered together (purple).

Table 2.2: Effective radiative forcing from 1750 to 2022. The latest radiative forcings estimates by the IGCC (Forster et al., 2024) are compared to the FaIR ensemble driven by all emissions, fossil fuels, and other activities. The second column differs from the sum of the third and fourth columns due to nonlinearities in FaIR (see Appendix). Median and 5th-95th interquantile ranges are shown.

Forcing	IGCC	FaIR	FaIR: Fossil Fuels	FaIR: Other Activities
Total	2.91 (2.17/3.62)	2.88 (2.29/3.46)	1.47 (1.07/1.90)	1.76 (1.29/2.16)
CO ₂	2.25 (1.98/2.52)	2.25 (1.98/2.52)	1.58 (1.38/1.79)	0.75 (0.65/0.85)
CH ₄	0.56 (0.45/0.67)	0.58 (0.47/0.69)	0.27 (0.21/0.32)	0.55 (0.44/0.66)
N ₂ O	0.22 (0.19/0.25)	0.22 (0.18/0.24)	0.04 (0.03/0.04)	0.19 (0.16/0.21)
Aerosols	-0.97 (-1.59/-0.39)	-0.93 (-1.54/-0.41)	-0.73 (-1.10/-0.32)	-0.25 (-0.77/-0.03)
Aerosol-cloud	-0.77 (-1.35/-0.23)	-0.73 (-1.33/-0.26)	-0.55 (-0.93/-0.19)	-0.23 (-0.76/-0.02)
Aerosol-radiation	-0.20 (-0.41/0.01)	-0.20 (-0.40/0.01)	-0.18 (-0.35/0.01)	-0.03 (-0.08/0.03)
Ozone	0.48 (0.24/0.71)	0.45 (0.22/0.65)	0.26 (0.12/0.39)	0.28 (0.14/0.43)
Halogens	0.41 (0.33/0.49)	0.40 (0.33/0.47)	0.00 (0.00/0.00)	0.40 (0.33/0.47)
Land-use albedo	-0.20 (-0.30/-0.10)	-0.20 (-0.30/-0.09)	0.00 (0.00/0.00)	-0.20 (-0.30/-0.09)
Particles on snow + ice	0.07 (-0.00/0.16)	0.06 (-0.00/0.14)	0.04 (-0.00/0.08)	0.03 (-0.00/0.06)
Stratospheric water vapor	0.05 (-0.00/0.10)	0.05 (0.00/0.11)	0.03 (0.00/0.05)	0.05 (0.00/0.11)

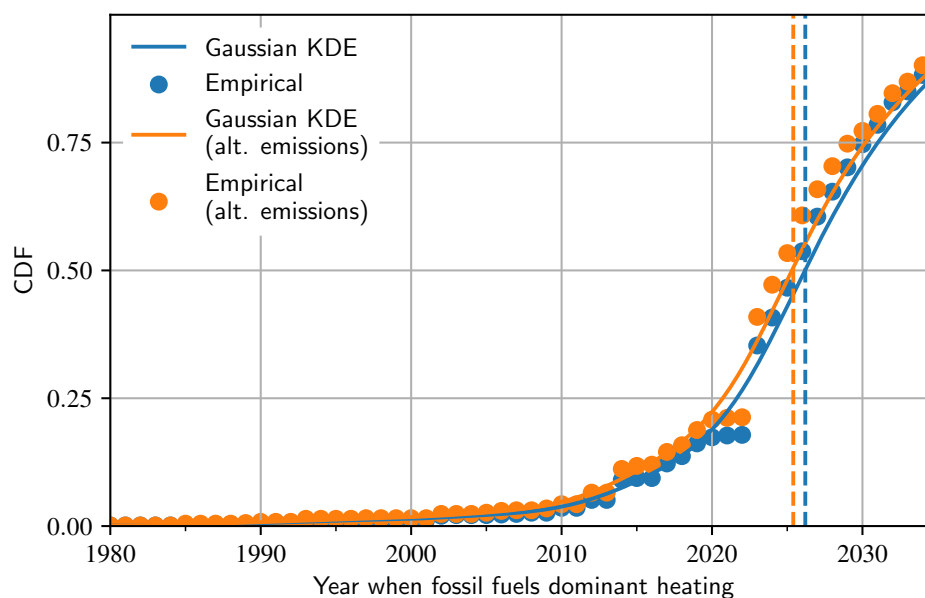


Figure 2.12: Cumulative distribution function (CDF) of the projected year when the heating from fossil fuels equals the heating from other activities in the FaIR ensemble, analyzed under two different emissions decompositions: ambiguous emissions assigned to other activities (blue) and ambiguous emissions assigned to fossil fuels (orange); see Table 1 for details. Both the empirical CDF (circles) and a Gaussian kernel density estimate (KDE; solid curves), fitted with bandwidth determined by Silverman’s rule, are shown. Differentiating this CDF yields the PDF in Figure 2.1. Dashed lines represent the median estimates for each decomposition. In both scenarios, it is likely ($>50\%$ chance) that fossil fuels will become the dominant source of heating within this decade.

Chapter 3

Emissions cessation yields multi-century cooling

Predicting the long-term warming commitment from past emissions is hindered by a lack of consensus amongst climate models. As a result, there is very low confidence that reaching net-zero emissions in the coming decades will stabilize the temperature beyond the 21st century. Here we provide a novel means of inferring the multi-century warming commitment directly from observations and established climate metrics. We consider the cessation of emissions of all greenhouse gases and aerosols, and quantify the post-emissions temperature change by comparing the disequilibrium in the carbon system to the disequilibrium in the ocean’s thermodynamics. Constraining these disequilibria yields a prediction of multi-century change of approximately -0.4°C if emissions presently cease. For future emissions pathways, similar constraints indicate multi-century cooling after the cessation of all emissions. Relative to the temperatures at net-zero, idealized theory and climate models of reduced and intermediate complexity predict a long-term decline of 0.7 to 1.0°C depending on the scenario. These findings build confidence that net-zero mitigation pathways yield declining temperatures beyond the 21st century.

3.1 Introduction

What happens to global temperatures after emissions cease? This question is of critical importance for climate change policy, since any warming after emissions halt needs to be accounted for in proposed carbon budgets (Palazzo Corner et al., 2023). This significance has led to the study of the “zero emissions commitment” (ZEC), or the temperature change that occurs after emissions cease.

The content of this chapter previously appeared in Tarshish et al. (2024).

A body of relatively recent work has found that ZEC is close to zero, a finding that appears to be relatively robust across models as well as emissions scenarios (Solomon et al., 2009; Matthews and Caldeira, 2008; Gillett et al., 2011; Zickfeld et al., 2013; Ehlert and Zickfeld, 2017; MacDougall et al., 2020). This has led to a central ZEC estimate of zero in the most recent Intergovernmental Panel on Climate Change (IPCC) assessment (Canadell et al., 2021, Section 5.5.2.2.4), as well as the assumption that $ZEC \approx 0$ for the purposes of carbon budgeting (Rogelj et al., 2019). Much of this work, however, has focused on the *multi-decadal* response to zero emissions.

On longer timescales, ZEC turns out to be more uncertain. Indeed, while assessing multi-decadal ZEC to be $0 \pm 0.2^\circ\text{C}$, the latest IPCC Assessment concludes with only *very low confidence* that their assessed carbon budgets yield climate stabilization beyond the 21st century (Canadell et al., 2021, Section 5.5.2.3). (In this article, $\pm$ signifies standard deviation, and $[-]$ indicates 5th-95th interquantile range.) This uncertainty presents an obstacle for political ambition to limit temperature rise indefinitely (Palazzo Corner et al., 2023). While the time-horizon of the Paris Agreement is ambiguous, such an indefinite limit is a plausible interpretation of the agreement’s “long-term temperature goal” of “[h]olding the increase in the global average temperature to well below 2°C above pre-industrial levels and pursuing efforts to limit the temperature increase to 1.5°C above pre-industrial levels” (par, 2015).

The uncertainty in centennial committed warming due specifically to CO_2 is illustrated in Figure 1, reproduced from a recent inter-comparison project (MacDougall et al., 2020, Figure 3b), showing climate model responses following the cessation of CO_2 emissions in an idealized experiment. If we restrict our analysis to the multi-decadal response, as in recent assessments (MacDougall et al., 2020; Lee et al., 2021b), the standard deviation across models 50 years after emissions cease is indeed 0.2°C . However, if we instead focus on the multi-century timescale, over which models substantially diverge, we find millennial temperatures with a range of 1.3°C and standard deviation of 0.4°C . While this more uncertain evolution lies beyond the end-of-century time-horizon typically spotlighted by the IPCC, these temperatures determine the fate of slowly-evolving components of the Earth system, with profound implications for glaciers, many ecosystems, and sea level rise.

Model disagreement on the long-term post-emissions response to CO_2 is perhaps not surprising, given that many fundamental quantities, including the equilibrium climate sensitivity, vary substantially across models. What is more perplexing, however, is that conventional climate metrics are only weakly correlated with ZEC (see MacDougall et al. (2020) and Supplementary Fig. 3.5). Consequently, the post-emissions response to CO_2 appears to be an emergent property of climate models, largely unconstrained by other lines of evidence. Another challenge to untangling long-term ZEC is that present-day forcing (Forster et al., 2023) is due to both long-lived CO_2 ($2.25 [1.98 - 2.52] \text{ W/m}^2$) as well as non- CO_2 forcings ($.67 [-0.01 \text{ to } 1.32] \text{ W/m}^2$). The large spread in the non- CO_2 forcing, stemming largely from the aerosol component in particular (Forster et al., 2023), makes the all-species ZEC more uncertain. Upon emissions cessation, most of this net heating from non- CO_2 forcing shrinks

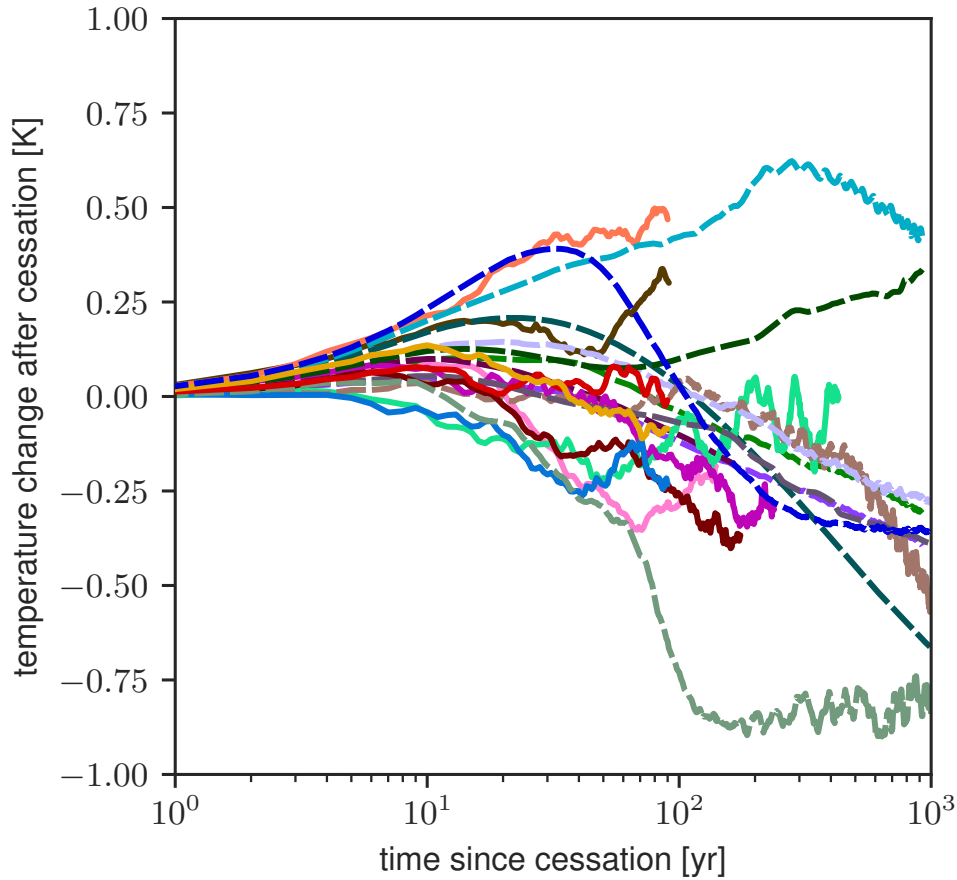


Figure 3.1: Projections of CO_2 committed warming diverge over time. Temperature change after CO_2 emissions of 1000 GtC as realized by full complexity (solid) and intermediate complexity (dashed) models participating in ZECMIP (MacDougall et al., 2020). Plot is reproduced from Figure 3b of MacDougall et al. (2020) with a logarithmic x-axis to emphasize inter-model divergence. Prior to zero emissions, CO_2 concentration increases at 1%/yr and emissions are inferred from the carbon budget. Refer to Fig. 3.2 for legend.

to zero on a decadal timescale (Supplementary Fig. 3.12) – a cooling effect which is expected to be superimposed on top of the CO₂-only response shown in Fig. 3.1. One might expect this to lead to a negative ZEC, but results are in fact mixed as to whether temperature rises or falls if emissions of all types cease. Several modeling studies, relying on the same simple climate model, do find an end-of-century cooling relative to the present-day of -0.2 to -0.3°C (Smith et al., 2019; Masson-Delmotte et al., 2018a; Dvorak et al., 2022). However, another analysis (Matthews and Zickfeld, 2012), using a more sophisticated model, finds that a similar cessation experiment yielded an end-of-century temperature $+0.2^{\circ}\text{C}$ above present-day, and an analytic model constrained by observations (Mauritsen and Pincus, 2017) also predicts committed temperature rise this century.

Here, we aim to bring long-term ZEC into sharper focus, using a hierarchy of models. First, by reasoning about the physics of a simple energy balance model, we develop an analytic theory that explains the climate model divergences in Fig. 3.1 in terms of established metrics of climate sensitivity and carbon uptake. We then constrain these metrics with multiple lines of evidence, yielding a prediction of multi-century *cooling* if present-day emissions cease, due to vanishing short-lived forcing and a waning temperature response to past CO₂ emissions. Generalizing to future emissions, we further demonstrate that similar constraints, derived from theory and intermediate-complexity models, also yield cooling after net-zero in the Shared Socioeconomic Pathways (SSPs), illustrated here with the widely-used climate model emulator FaIR (Leach et al., 2021a). Together, these results point to multi-century cooling as a robust response to emissions cessation, allowing for more confident reasoning about temperature stabilization beyond the 21st century.

3.2 Physical basis of committed warming

The temperature evolution after CO₂ emissions cease is qualitatively understood to depend on the competition between the declining heat source (the atmospheric abundance of CO₂) and the declining heat sink (the cold deep ocean) (Matthews and Caldeira, 2008; Solomon et al., 2009). More than half of historical CO₂ emissions have already been removed from the atmosphere by land and ocean uptake (Friedlingstein et al., 2022). Absent future emissions, airborne anthropogenic CO₂ is expected to be roughly halved again as the land, ocean, and atmospheric carbon reservoirs equilibrate in the coming centuries (Archer et al., 2009; Matthews and Weaver, 2010). Over similar timescales, the deep ocean warms, diminishing its ability to cool the surface ocean via mixing. This decline in the oceanic heat sink warms the surface, countering the cooling from declining CO₂ concentrations (Matthews and Caldeira, 2008; Solomon et al., 2009; MacDougall et al., 2020).

As evidenced by Figure 3.1, the relative magnitudes of these competing effects are not consistent across models, yielding a heterogeneous set of zero emissions commitments (ZEC) quantified (Hare and Meinshausen, 2006) as the difference between the final long-term tem-

perature anomaly T_f and the temperature anomaly at the time emissions cease T_{ze} ,

$$\text{ZEC} = T_f - T_{ze}. \quad (3.1)$$

Anomalies are taken with respect to the preindustrial era, and long-term anomalies refer to the carbon and thermal quasiequilibria that emerge after substantial deep-ocean mixing over the following centuries. In particular, we set T_f equal to the temperature 1000 years after cessation, an approximate measure of this long-term state, in which oceanic, land, and atmospheric carbon are equilibrated. To reach full equilibrium, atmospheric carbon must also equilibrate with ocean sediments and silicate rocks, which on timescales of 10 to 100 millennia, restores pCO_2 to preindustrial values (Archer et al., 2009).

Consider the magnitude of ZEC relative to the temperature anomaly at cessation,

$$\frac{\text{ZEC}}{T_{ze}} = \frac{T_f}{T_{ze}} - 1, \quad (3.2)$$

In the final state, $T_f = F_f/\lambda$, where F_f is the final radiative forcing and λ is the equilibrium climate feedback parameter. Let T_{eq} be the hypothetical equilibrium $T_{eq} = F_{ze}/\lambda$ that would arise if the forcing were constant at the zero-emissions value F_{ze} . In terms of these variables,

$$\frac{\text{ZEC}}{T_{ze}} \approx \frac{F_f/F_{ze}}{T_{ze}/T_{eq}} - 1, \quad (3.3)$$

The ratio F_f/F_{ze} represents the *forcing* disequilibrium at cessation due to the transient presence of short-lived pollutants and CO_2 that eventually will be taken up by the land and ocean. Likewise, T_{ze}/T_{eq} quantifies the *thermal disequilibrium* in the transient response due to the not-yet-warmed deep ocean.

To make progress, we constrain the thermal disequilibrium by appealing to the two-box model of climate (Gregory, 2000; Held et al., 2010; Jeevanjee, 2018). This model demonstrates that the surface ocean comes into quasi-equilibrium with the forcing on a timescale of a few years. In this quasi-equilibrium state, the radiative heating of the surface ocean is balanced by cooling to space and the deep ocean. The thermal disequilibrium between the surface and deep ocean is additionally fixed to a constant if the effective temperature of the deep ocean has not changed since pre-industrial (Eqn. 16 in Appendix). When fit to comprehensive models, the effective deep-ocean temperature in the two-box model evolves on a characteristic response timescale of ~ 200 years (Geoffroy et al., 2013). For the historical case, radiative forcing has rapidly increased only over the last 50 years (Forster et al., 2023), meaning that the two-box model of the present-day climate corresponds to a surface ocean in quasi-equilibrium with a preindustrial, cold deep ocean (Held et al., 2010). As shown in Supplementary Figure 3.6, surface quasi-equilibrium with a pre-industrial deep ocean fixes T_{ze}/T_{eq} to be relatively constant over the historical period. The value of this constant thermal disequilibrium can be inferred by appealing to other scenarios in which the surface ocean is in quasi-equilibrium and the effective deep ocean temperature has not

substantially increased. To connect to established climate metrics, we consider the time of doubling (year 70) following a 1%/yr increase in CO_2 . In this case, the thermal disequilibrium is equal to the ratio of the temperature anomaly at doubling, known as the transient climate response (TCR), to the final temperature if forcing were held constant at doubling, i.e., the equilibrium climate sensitivity (ECS).

If in a more general scenario the effective deep-ocean temperature anomaly can still be neglected at the time of zero-emissions, then the thermal disequilibrium must still be constrained to be TCR/ECS , yielding

$$\frac{\text{ZEC}}{T_{ze}} \approx \frac{F_f/F_{ze}}{\text{TCR}/\text{ECS}} - 1. \quad (3.4)$$

Given the extensive body of research constraining TCR and ECS with multiple lines of evidence (Sherwood et al., 2020; on Climate Change, 2014), the above formula will enable us to constrain the post-emissions response in light of our broader understanding of climate change.

The remaining degree of freedom is the forcing disequilibrium. If CO_2 is the only forcing (as in Figure 1), the forcing disequilibrium can be directly related to the disequilibrium in the carbon system. Expressing the logarithmic CO_2 forcing in terms of the cumulative emissions C_{emit} and the remaining airborne fractions A_{ze} and A_f in the zero-emissions state and final state, respectively, yields

$$\frac{F_f}{F_{ze}} = \frac{\log(1 + A_f C_{\text{emit}}/C_{\text{pre}})}{\log(1 + A_{ze} C_{\text{emit}}/C_{\text{pre}})} \quad (3.5)$$

where $C_{\text{pre}} \approx 590 \text{ GtC}$ is the pre-industrial mass of atmospheric carbon (Friedlingstein et al., 2022). While we retain this full logarithmic expression when computing ZEC in future sections, we linearize the logarithms here to build intuition. If $C_{\text{emit}} \ll C_{\text{pre}}/A_{ze} \approx 1500 \text{ GtC}$, estimated with the present-day airborne fraction (Friedlingstein et al., 2022), then expanding the logarithms yields $F_f/F_{ze} \approx A_f/A_{ze}$. The post-emissions response is then determined by simply comparing the *carbon* disequilibrium (A_f/A_{ze}) to the thermal disequilibrium:

$$\frac{\text{ZEC}}{T_{ze}} \approx \frac{A_f/A_{ze}}{\text{TCR}/\text{ECS}} - 1. \quad (3.6)$$

This provides a quantitative basis for the finding in climate models that the relative degrees of disequilibrium in the carbon and thermal responses influence ZEC (Ehlert and Zickfeld, 2017; Williams et al., 2017; MacDougall et al., 2020). In the context of an idealized impulse response model of the carbon cycle, Allen et al. (2022a) derives a related analytic expression for ZEC and concludes that $\text{ZEC} = 0$ if the realized warming fraction ($\approx \text{TCR}/\text{ECS}$) equals the fraction of CO_2 -induced forcing that persists indefinitely ($\approx A_f/A_{ze}$). If $\text{TCR}/\text{ECS} < A_f/A_{ze}$, then the thermodynamics are further from equilibrium than the carbon system is, in which case the impact of the declining ocean heat sink dominates over the reduction in CO_2

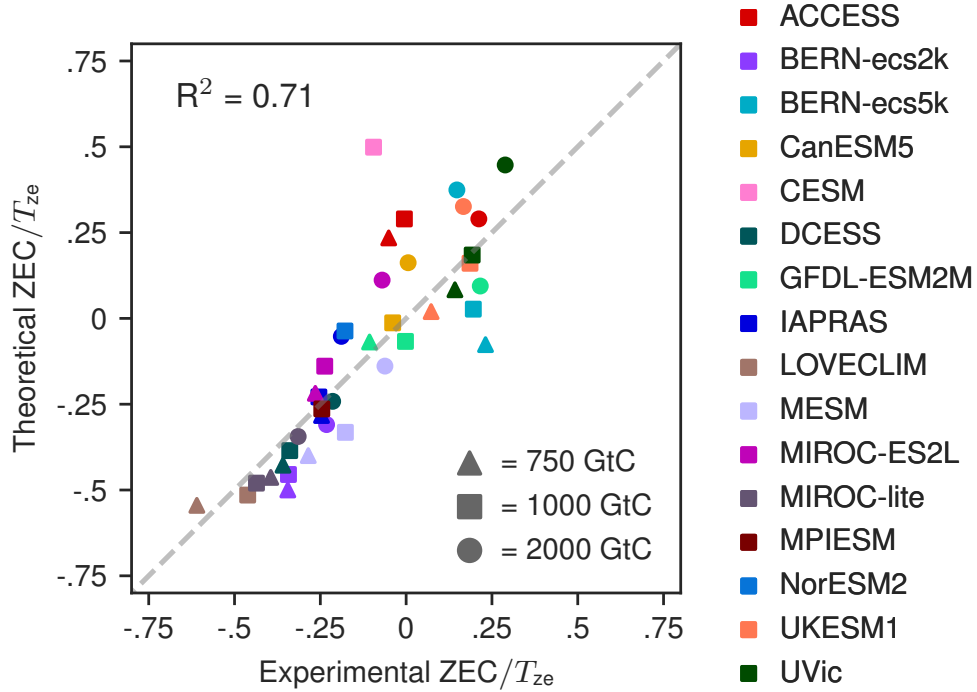


Figure 3.2: Model variation explained by theory. Fractional change in the temperature anomaly after emissions cessation as realized by EMIC simulations versus the theoretical formula (Eq. 3.4) for cumulative emissions of 750 GtC (triangles), 1000 GtC (squares), and 2000 GtC (circles). Prior to zero emissions, models are forced by a 1%/yr increase in CO_2 concentration. For CMIP6 models, these results do not represent long-term committed warming, as simulations terminated before equilibrium.

forcing, yielding positive ZEC. Conversely, if the carbon system is further from equilibrium than the thermodynamics, the decline in CO_2 forcing dominates, yielding negative ZEC.

Assuming surface quasi-equilibrium with a cold deep ocean at the time of cessation, scenario dependence of ZEC only enters via the carbon disequilibrium: A_{ze} is expected to be higher with faster emissions (Raupach, 2013), and A_f depends on the magnitude of cumulative emissions (Goodwin et al., 2007a; Eby et al., 2009; Archer et al., 2009). Carbon uptake also depends on temperature, and while the strength of carbon-climate feedbacks varies across models, their presence increases A_f (Friedlingstein et al., 2006; Arora et al., 2013).

While neglecting the effective deep-ocean temperature anomaly accurately approximates the present-day climate response, this temperature will grow over the following century, as the ocean advances towards thermal equilibrium. The resultant increase in T_{ze}/T_{eq} depends

on the particular SSP, ranging from 10% to 20%, with higher values associated with more stable forcing pathways (see Supplementary Figure 3.6).

3.3 Theory explains variation across climate models

We analyze ZECMIP experiments (MacDougall et al., 2020) in which the atmospheric CO_2 concentration increases at a rate of 1%/yr until the diagnosed cumulative emissions reach 750, 1000, and 2000 GtC. The temperature evolution for the 1000 GtC case is shown in Fig. 3.1, and temperature and carbon evolutions for all cases can be found in Supplementary Figures 3.7 and 3.8. The ZECMIP ensemble contains Earth system models of intermediate complexity (EMICs) and full-complexity models participating in the Coupled Model Intercomparison Project Phase 6 (CMIP6) (Eyring et al., 2016). Due to computational constraints, the CMIP6 models were not run to equilibrium. As apparent in Fig. 3.1, most CMIP6 simulations terminate less than 200 years after emissions cessation. For these models, we adapt the idealized formalism to predict ZEC at the end of the simulation, rather than predicting the long-term equilibrium ZEC (see Appendix).

We use the ECS and TCR values listed in MacDougall et al. (2020) for each model and diagnose A_{ze} and A_f from the simulated CO_2 time series (see Supplementary Figure 3.9). The resultant theoretical estimates of ZEC are compared to the experimental values in Fig. 3.2. We find a close match between the normalized ZEC anticipated by the analytic model (Eq. 3.4) and the normalized ZEC realized in the ESM simulations. The theoretical formula yields values that are, on average, within 0.12 of the experimental ZEC/T_{ze} . With a Pearson’s coefficient of $R^2 = 0.71$, there is a strong correlation between the analytic and ESM results. Comparison to the ZECMIP data therefore builds confidence that Eqn. 3.4 broadly captures the climate’s response to emissions cessation. In light of the ZEC formula, we would expect historical ZEC and 1%/yr to differ due to the scenario-dependence of the airborne fraction. Faster emissions in the 1%/yr case yield a higher A_{ze} of approximately 0.6 (see Supplementary Figure 3.9) compared to the historical airborne fraction of roughly 0.4 (Friedlingstein et al., 2022). Beyond differences in airborne fraction, ZECMIP is an ensemble of opportunity, and thus the particular distribution of TCR/ECS values realized in ZECMIP differs from the best-estimate based on multiple lines of evidence. In addition, the ZECMIP experiments neglect the substantial contributions from aerosols and short-lived climate pollutants. These myriad differences raise concerns around the past practice (Solomon et al., 2009; Ehlert and Zickfeld, 2017; MacDougall et al., 2020) of using model responses to CO_2 1%/yr forcings to infer ZEC for historical emissions. Below, we address these issues by directly applying the ZEC formula to the historical case for the cessation of all species, using observations and best estimates of the thermal disequilibrium.

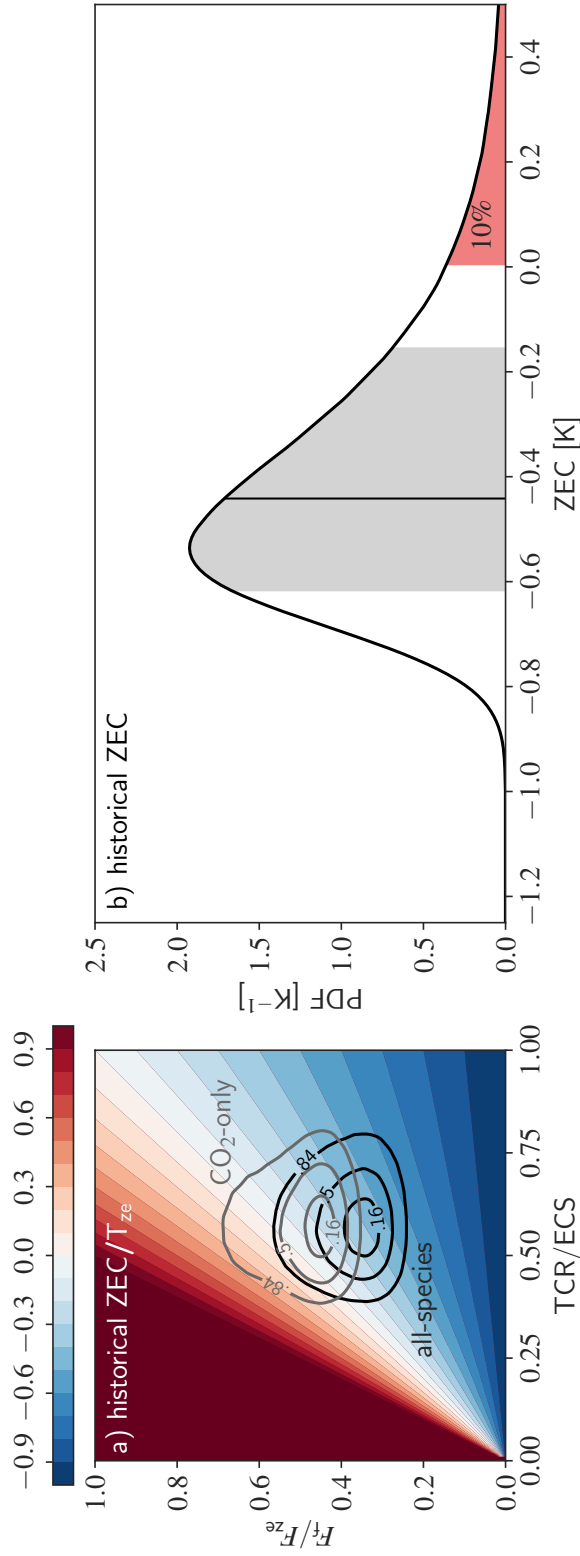


Figure 3.3: Temperature declines absent future emissions: (a) the fractional change in long-term temperature given zero future emissions (color) in terms of the present-day thermal disequilibrium (x-axis) and forcing disequilibrium (y-axis). Contours contain 16%, 50%, and 84% of the probability mass given constrained estimates of these disequilibria for only historical CO₂ emissions (grey) and all historical emissions (black). Sampling the all-species distributions yields the resultant probability distributions for ZEC (b) with the 68% confidence interval of -0.63 to -0.15 K shaded grey.

3.4 Present-day committed warming

Evidence for a negligible deep-ocean temperature anomaly in the two-box representation of present-day climate comes from simulations (Held et al., 2010; Winton et al., 2020) in which the forcing is abruptly zeroed and the temperature rapidly returns to near pre-industrial levels, and from energy balance analysis (see Appendix). These findings support approximating the present-day thermal disequilibrium with TCR/ECS.

The IPCC AR6 central assessment for ECS is 3.0 K with a very likely (i.e., 5th-95th interquantile) range of [2.0-5.0] K (Forster et al., 2021b). This estimate synthesizes process-based understanding, warming over the instrumental record, paleoclimate, and emergent constraints (Sherwood et al., 2020). Likewise, the combined assessment of TCR is 1.8 [1.2-2.4] K. We take uncertainty in ECS to be skew-normally distributed and TCR uncertainty to be normally distributed. Based on theory and modeling, we expect TCR and ECS to co-vary (Nijse et al., 2020). To estimate TCR/ECS, we co-variantly sample the marginal distributions (Iman and Conover, 1982) such that the Monte Carlo results reproduce the correlation coefficient ($R = 0.78$) between TCR and ECS found in CMIP6 (Meehl et al., 2020), yielding $\text{TCR/ECS} = 0.58 [0.43 - 0.76]$.

To make contact with prior work, we first consider committed warming for the CO_2 -only case. Both C_{emit} (680 GtC) and A_{ze} (0.43) are calculated from reconstructions of historical emissions (Friedlingstein et al., 2022). The remaining necessary input to compute $F_{\text{f}}/F_{\text{ze}}$ (via Eqn. 3.5) is A_{f} , which cannot be directly measured. We infer A_{f} for historical emissions from the airborne fractions realized at year 1000 in the long-running ZECMIP experiments (see Supplementary Figure 3.14). Monte Carlo sampling the ZECMIP models and extrapolating from per-model linear fits to the variation of A_{f} across the differing C_{emit} experiments yields a final airborne fraction estimate of 0.17 [0.16 - 0.26]. ZECMIP models find that the ocean dominates long-term carbon uptake (see Supplementary Figure 3.11). In fact, the ZECMIP ensemble-mean A_{f} is broadly consistent with a back-of-the-envelope calculation given only ocean uptake: assuming that C_{emit} is divided between the atmosphere and ocean according to the preindustrial partitioning of carbon between the atmosphere and the buffered ocean inventory yields

$$A_{\text{f}} \approx \frac{C_{\text{pre}}}{C_{\text{pre}} + \text{DIC}/R} \approx 0.2, \quad (3.7)$$

where $R \approx 10$ is the Revelle buffer factor (Revelle and Suess, 1957) and $\text{DIC} \approx 3 \times 10^4$ GtC is the total mass of dissolved inorganic carbon (Goodwin et al., 2007a; Williams et al., 2016).

For contemporary uptake, the terrestrial biosphere also plays an important role due to its relatively fast equilibration timescales with the atmosphere. For this same reason, after emissions, as the atmospheric CO_2 declines from ocean uptake, CO_2 fertilization of photosynthesis also declines, yet decomposition maintains, resulting in a final equilibrium with only ~ 100 GtC of land uptake in ZECMIP models. The land also robustly becomes a net source of CO_2 once the atmospheric concentration begins to decline in full-complexity

ESMs; however, the regional pattern of land-atmosphere CO₂ exchange varies widely across models (Koven et al., 2022).

As shown in Fig. 3.3a, the ZECMIP A_f results yield a forcing disequilibrium of $F_f/F_{ze} = 0.48 [0.38 - 0.68]$, less than the central estimate of the thermal disequilibrium. This produces a negative ZEC/ T_{ze} for CO₂-only of $-0.16 [-0.43 \text{ to } 0.3]$, with a 75% probability of negative values. Given that we expect the 1%/yr case to exhibit more cooling from A_{ze} considerations, this weak cooling for the historical case is consistent with the majority of millennial-scale EMIC results shown in Figure 3.1. Furthermore, the analytical framework utilized here allows us to explain the positive long-term ZEC exhibited by some outlier EMICs (see Appendix), narrowing some of the apparent uncertainty implied by Figure 3.1. For instance, warming in the model with the largest fractional ZEC (Bern-ecs5k) is due to this model’s TCR/ECS value falling beneath the 5-95% confidence interval of the AR6-based estimate.

Generalizing to the case of all historical emissions, we note that the all-species forcing estimate is $F_{ze} = 2.91 [2.19 - 3.63]$ W/m², larger than the CO₂-only forcing estimate of $2.25 [1.98 - 2.52]$ W/m² (Forster et al., 2023). However, both cases share nearly identical F_f as CO₂ is the dominant forcing on millennial timescales (see Supplementary Figure 3.12). Therefore, the all-species case exhibits a greater degree of forcing disequilibrium, yielding a larger relative decline in forcing – a smaller value of F_f/F_{ze} – and thus more cooling after emissions end. A higher T_{ze} for the all-species case, linked to the estimated additional 0.65 W/m² of non-CO₂ forcing (see Fig. 3.3a), is responsible for this greater cooling. Our finding that all-species ZEC is less than the CO₂-only ZEC runs counter to prior literature, which estimated that both cases shared equivalent long-term responses, but did so on the basis of simulations lasting only two centuries after emission (Matthews and Zickfeld, 2012; MacDougall et al., 2020). By Monte Carlo sampling the all-species forcing disequilibrium, we compute an all-species ZEC/ $T_{ze} = -0.34 [-0.58 \text{ to } 0.10]$, which can be attributed in roughly equal parts to the disappearance of short-lived forcing and a temperature response to CO₂ that declines over time.

So far we have examined the fractional temperature change ZEC/ T_{ze} , linking this post-emissions response to familiar metrics, elucidating its physical origin, explaining the varying climate model results, and finding a robust prediction of cooling absent future emissions. To understand the implications for specific temperature targets, however, we now pivot to constraining the absolute magnitude of ZEC. Based on observations, we take the current human-attributable warming to be 1.3 [1.1 - 1.5] K (Haustein et al. (2017) updated to present-day at www.globalwarmingindex.org). Multiplying the prior ZEC/ T_{ze} estimate via this T_{ze} would exaggerate the role of uncertainty associated with the present-day temperature, since ZEC/ T_{ze} already contains redundant T_{ze} uncertainty via the TCR distribution.

Instead, we estimate ZEC by separately computing the true additional degree of freedom, T_f , which was already implicitly constrained by A_f and ECS. Making this constraint explicit, we compute

$$T_f = \frac{F_f}{\lambda} = \frac{F_f}{F_{2\times}} \text{ECS} = \log_2 \left(1 + \frac{A_f C_{\text{emit}}}{C_{\text{pre}}} \right) \text{ECS} , \quad (3.8)$$

where we applied the definition $ECS = F_{2\times}/\lambda$ and substituted in for F_f with the logarithmic CO_2 forcing expression. Monte Carlo sampling $ZEC = T_f - T_{ze}$, given the ZECMIP A_f distribution (see Supplementary Figure 3.14), and assuming historical warming and ECS covary with the same correlation ($R = .73$) found in CMIP6 models (Tokarska et al., 2020) yields the distribution plotted in Fig. 3.3(b). We find that all-species $ZEC = -0.45$ $[-0.72$ to $0.11]$ K, indicating substantial cooling in the absence of future emissions. Uncertainty in ZEC stems primarily from two sources: the spread in F_{ze} due to weak constraints on aerosol forcing, and the long tail of the ECS distribution.

Even with these uncertainties, past emissions very likely ($> 90\%$ probability) commit the long-term temperature to values less than the present-day anomaly. Despite this moderate temperature decline, continued oceanic uptake of heat and carbon is expected to raise sea levels and acidify the ocean (Solomon et al., 2009; on Climate Change, 2014). Glaciers and ice sheets will also continue to melt as they equilibrate to long-term warming. Thus, the switch from historical warming to post-emissions cooling does not reverse the intensifying trend of many climate change impacts.

3.5 Committed warming in the SSPs

Given the understanding developed here as to why climate models diverge on ZEC , and the demonstration that narrowing this uncertainty with multiple lines of evidence robustly predicts $ZEC < 0$ for the historical emissions trajectory, we now investigate if future emissions pathways also exhibit post-emissions cooling. In particular, we revisit the question of whether the AR6 carbon budget and its associated mitigation pathways stabilize the temperature beyond the 21st century. As illustrated in Appendix Figure 2, the thermal disequilibrium in future scenarios can stray from the quasi-equilibrium TCR/ECS approximation as the deep ocean responds, necessitating a different approach to constraining ZEC in the SSPs than for historical. Here we take a carbon-budgeting approach to infer ZEC for general future emissions.

The AR6 carbon budget relies on an emergent feature of climate models: transient temperature change is nearly linear in cumulative CO_2 emissions (Matthews et al., 2009). The proportionality between transient warming and cumulative emissions is known as the Transient Climate Response to Cumulative Emissions (TCRE), and appears to be approximately scenario-independent (Canadell et al., 2021). In AR6, TCRE is employed to constrain the temperature at the time when net-zero CO_2 emissions are achieved (sometime in the mid to later half of the century in aggressive mitigation scenarios) (Meinshausen et al., 2020). AR6 assessed that the peak temperature can deviate from this T_{ze} on multi-decadal timescales due to scenario-dependent non- CO_2 forcings ($\pm 0.1^\circ C$) and the short-term post-emissions response ($\pm 0.2^\circ C$). Since estimates of these contributions are centered on zero, we approximate T_{ze} as the peak temperature, determined by the AR6 carbon budget and plotted in Figure 4a (red curve).

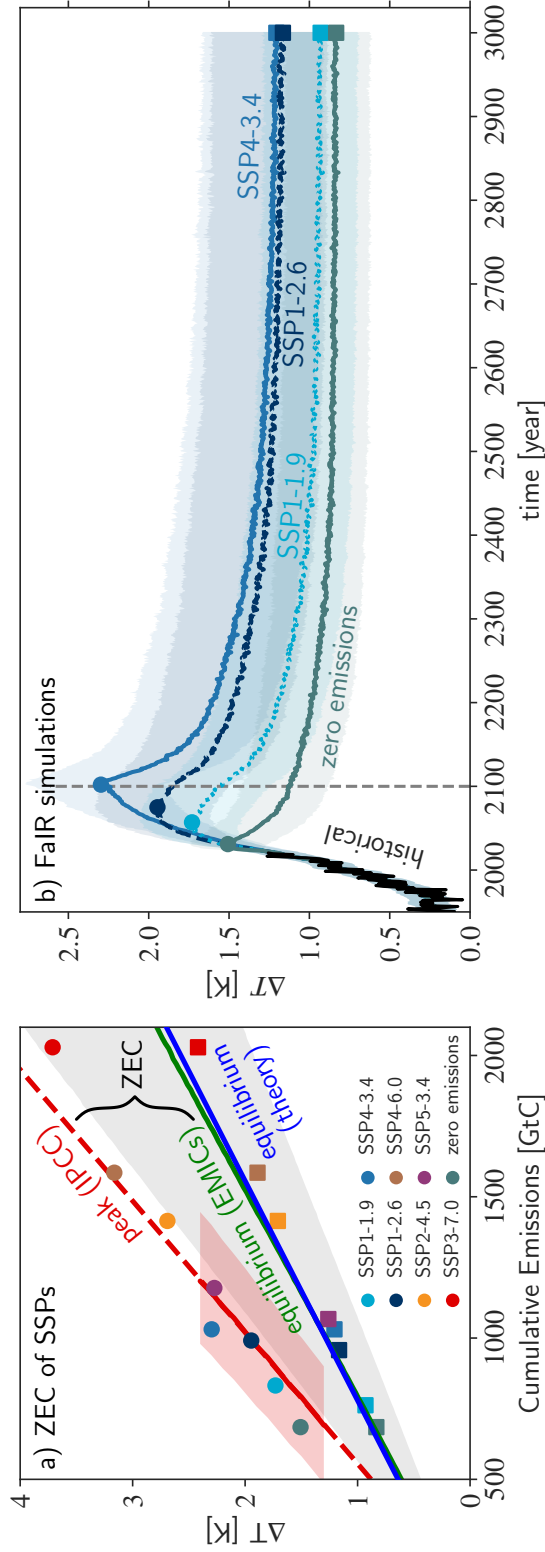


Figure 3.4: Substantial cooling after net-zero in the SSPs. a) equilibrium warming inferred from EMICs (green) and theory (blue) compared to the AR6 peak warming carbon budget (red) and the ensemble-mean FaIR results for the SSPs (markers). b) millennial FaIR simulations of the temperature response to zeroing emissions now (teal) or 2100 after following SSP mitigation pathways.

Here we point out that the prior T_f constraint (Eqn. 3.8) for historical emissions generalizes to arbitrary emissions and enables computation of the *carbon budget for equilibrium warming*. To find A_f for general C_{emit} , we again rely on the linear fits diagnosed in the ZECMIP models. Monte Carlo sampling Eqn. 3.8 given these A_f results and the AR6 ECS distribution yields the carbon budget for equilibrium warming (green line) in Figure 3.4a.

Alternatively, if A_f is constrained by an idealized theory (Goodwin et al., 2007a) of equilibrium ocean uptake at fixed temperatures and circulation, nearly equivalent results are found (blue line) as shown in Figure 3.4a. This close match suggests compensation between the second-order effects of equilibrium land uptake and carbon-climate feedbacks, both neglected in the theory. In this idealized ocean-atmosphere system (see Appendix), the final temperature can be analytically derived (Goodwin et al., 2015),

$$T_f = \frac{C_{\text{emit}} \text{ ECS}}{\log(2) (C_{\text{pre}} + \text{DIC}_{\text{sat}}/R)}, \quad (3.9)$$

where both here and in the EMIC results, the final atmospheric carbon mass increases super-linearly with C_{emit} (see Equation 23), which when composed with the logarithmic dependence of the forcing, linearizes the relationship between T_f and C_{emit} .

For a general emissions pathway, equilibrium ZEC can be simply read off Figure 3.4a as the vertical distance between the equilibrium and peak lines evaluated at a given cumulative future emissions. The key finding is that equilibrium ZEC is negative for all future scenarios, meaning that pathways compliant with the AR6 carbon budgets successfully stabilize the long-term temperature to values *beneath* their respective mid-century peak temperatures. Therefore, the AR6 carbon budgets for 1.5°C and 2°C, in particular, achieve the goals of the Paris Accords beyond the 21st century.

For the 2°C goal, compliant pathways are expected to exhibit a multi-century temperature decline of roughly $\text{ZEC} = -0.7^\circ\text{C}$ back to near present-day temperature ($\sim 1.3^\circ\text{C}$) after net-zero. Here we illustrate the time evolution of this cooling by performing SSP simulations with the simple climate model FaIR (see Appendix). Projections utilize a 841-member ensemble of FaIR model parameter choices (Smith, 2024) tuned to reproduce AR6 uncertainty estimates (see Appendix for details). For the SSPs, Figure 3.4a demonstrates that millennial temperatures realized in FaIR compare favorably with the EMIC and ocean-atmosphere theory results. Consistent estimates across these multiple rungs of the model hierarchy support the robustness of the findings.

The time evolution of several mitigation pathways are plotted in Figure 3.4b. See Appendix Figure 3.13 for the remaining scenarios. Here we emphasize scenarios with 21st century temperatures near the Paris targets. Emissions of all species are prescribed up to 2100 and then zeroed. All of the plotted mitigation trajectories exhibit substantial cooling in the centuries following emissions cessation and reach equilibrium temperatures beneath the Paris targets. The importance of non- CO_2 forcings in determining ZEC is highlighted by comparing SSP1-2.6 and SSP4-3.4, which cumulatively emit roughly the same amount of

CO₂, yet exhibit peak temperatures differing by nearly half a degree due to higher emissions of short-lived GHG emissions in SSP4-3.4.

For completeness, we also evaluate historical ZEC by driving the FaIR ensemble with historical emissions up to present-day and zero future emissions. The ensemble possesses a millennial-scale ZEC of -0.47 [-0.86 to 0.06] K, in remarkably close agreement with the constrained theoretical estimate (-0.45 [-0.72 to 0.11] K). Prior to this gradual decline, the temperature has a local maximum, peaking at +0.23 [-0.13 to 0.58] K above present-day. Temperatures initially rise due to the immediate loss of aerosols (see Appendix, Fig. 3.12). In subsequent decades, the disappearance of methane and ozone anomalies and natural CO₂ uptake lowers the forcing, yielding temperatures close to present day. Prior simulations of the all-species ZEC also observed this multi-decadal peak (Matthews and Zickfeld, 2012; Masson-Delmotte et al., 2018a; Tanaka and O’Neill, 2018; Dvorak et al., 2022), but have not projected the temperature far enough into the future to resolve the subsequent multi-century cooling.

This analysis complements two recent studies of the multi-decadal response to net-zero emissions (Jenkins et al., 2022; Dvorak et al., 2022). After net-zero was reached in 2050 following a 1.5°C stabilization pathway in FaIR, Jenkins et al. (2022) found a likely temperature decline of -0.1°C over the subsequent decades; as a consequence, weakly net-positive emissions of $\sim .5$ GtC/yr after 2050 were shown to be consistent with flat multi-decadal temperatures. The equilibrium results derived here contextualize this finding and emphasize that long-term temperature stability enforces a strict limit on cumulative future emissions. Dvorak et al. (2022) explored the implications of carbon budgeting for peak versus end-of-century temperature. These budgets were estimated from a set of cessation experiments in which FaIR was driven by SSP emissions up to a cessation year, varying from 2021 to 2080, and the post-emissions temperature was projected to 2100. (Dvorak et al., 2022). Dvorak et al. (2022) found empirically that the end-of-century temperatures were beneath peak temperatures, and concluded that budgeting for the 2100 temperature increases allowable future emissions. The equilibrium carbon budget derived here for 1.5 °C and 2°C are $\sim 70\%$ larger than the end-of-century budgets of ref (Dvorak et al., 2022).

Note that our estimates of equilibrium warming neglect thawing permafrost. Permafrost processes are uncertain and not yet represented in most CMIP6 models—AR6 estimates 18 [3 - 41] GtC of permafrost CO₂ emissions per degree of warming by 2100 (Forster et al., 2021b). However, a recent model inter-comparison project (McGuire et al., 2018) showed that models over the long-term on average *gain* permafrost soil carbon under RCP4.5 extended to 2300 due to vegetation growth, with changes ranging from a 66 GtC loss to a 70 GtC gain. These findings suggest that the net effect of permafrost thaw is unlikely to yield a first-order decrease to the equilibrium carbon budget.

Our estimates also assume that land-use emissions cease yet anthropogenic land use persists. If instead, land use ceases altogether, there will be additional carbon uptake from ecosystem restoration. If restoration over the next millennium is crudely estimated to draw down all of the historical land-use emissions (205 ± 60 GtC (Friedlingstein et al., 2022)),

then the final airborne emissions are reduced by a factor of approximately 5/7, lowering the prior estimate of all-species ZEC for historical from -0.45 $[-0.72$ to $0.11]$ K to -0.70 $[-0.89$ to $-0.38]$ K.

3.6 Discussion

Constraining the post-emissions response with idealized theory, metrics informed by multiple lines of evidence, and EMIC ensembles indicates long-term cooling after emissions cease for both historical and the SSPs. AR6 concluded with *very low confidence* that their assessed carbon budget stabilized the climate beyond the 21st century. Our findings build confidence that the AR6 carbon budget yields temperature trajectories that peak near the time of net-zero in the later half of this century and significantly decline over the subsequent centuries.

There would be little ambiguity about the Paris Accords’ “long-term temperature goal” if the temperature were instead roughly constant after net-zero in these mitigation pathways. However, the multi-century cooling identified here, prompts further consideration of the meaning of long-term. Our aim is not to suggest that the equilibrium temperature is a more natural policy target. The choice to target transient versus committed warming for mitigation requires a values-based judgement beyond the scope of this paper, and science itself. This policy choice does, however, have direct implications for whether short-lived GHG or long-lived CO₂ are prioritized for mitigation (Dreyfus et al., 2022; Allen et al., 2022b). In either case, the material differences between peak and multi-century temperatures identified here call for a sharpening of the policy discourse.

Appendix

Constraining the thermal disequilibrium

To probe the thermal disequilibrium, we use a simple two-box model (Gregory, 2000; Held et al., 2010; Armour, 2017; Jeevanjee, 2018). The lower box represents the deep ocean, and the upper box combines the ocean’s mixed layer and the atmosphere into a surface box. The internal energy of the surface box is dominated by the mixed layer, so we refer to the upper box simply as the surface ocean. Let T and T_d be the temperature deviation from the preindustrial era of the ocean’s surface layer and deep interior, respectively. The energy budget of the upper box is approximated as

$$C_s \frac{dT}{dt} = F - \lambda T - \lambda'(T - T_d) - \gamma(T - T_d), \quad (10)$$

where C_s is the mixed layer’s heat capacity (per area with units of J/K/m²), F is the radiative forcing (units of W/m²), and γ is the heat transfer coefficient between the two ocean layers (units of W/m²/K). The equilibrium climate feedback, which incorporates all

feedback processes, is parameterized as λT , where λ is the equilibrium change in the top-of-atmosphere outgoing radiation per change in surface temperature (in units of $\text{W}/\text{m}^2/\text{K}$). Climate models find that feedbacks have some dependence on the spatial pattern of warming (e.g. Rose et al., 2014; Armour, 2017; Andrews et al., 2022). In particular, ocean heat uptake delays warming in the Southern Ocean and eastern tropical Pacific, yielding cloud and ice-albedo feedbacks that differ between the transient and equilibrium states. Here λ' represents the transient deviation due to this pattern effect (Armour, 2017). (To make contact with the ocean heat uptake efficacy factor ϵ , note that $\lambda' = (\epsilon - 1)\gamma$).

The deep ocean's energy is governed by

$$C_d \frac{dT_d}{dt} = \gamma(T - T_d), \quad (11)$$

where C_d is the deep ocean's heat capacity. The box model's evolution consists of a fast and slow response to forcings (Held et al., 2010), which we will briefly illustrate here. Consider applying a step function forcing to the surface ocean. Initially $T_d = 0$, and the forcing excites a surface layer response with timescale,

$$\tau_f = \frac{C_s}{\gamma + \lambda + \lambda'}. \quad (12)$$

Over timescales much larger than τ_f , the tendency term in Eq. 10 is small, yielding a mixed layer in quasi-equilibrium at temperature

$$T \approx \frac{F + (\gamma + \lambda')T_d}{\lambda + \lambda' + \gamma}. \quad (13)$$

In quasi-equilibrium, the deep layer budget is reduced to

$$\frac{dT_d}{dt} = -\frac{T_d}{\tau_s} + \frac{\gamma F}{(\lambda + \lambda' + \gamma)C_d}, \quad (14)$$

where we have defined the deep ocean's slow timescale of response,

$$\tau_s = \frac{(\lambda + \lambda' + \gamma)C_d}{\lambda\gamma}. \quad (15)$$

Climate models find timescales of $\tau_f \sim 5$ years and $\tau_s \sim 200$ years (Gregory, 2000; Oliv   et al., 2012; Geoffroy et al., 2013). As shown in Appendix Figure 2, for present-day historical forcing, as well as near the time of doubling (year 70) in a 1%/yr increase in CO_2 experiment, the deep-ocean response is an order of magnitude smaller than the surface response, given model parameters fit to the CMIP5 ensemble-average (Geoffroy et al., 2013).

If we set $T_d = 0$, Eqn. 13 indicates that the ratio,

$$\frac{F}{T} = \lambda + \lambda' + \gamma, \quad (16)$$

is fixed to a constant. Let T_{eq} be the equilibrium warming $T_{\text{eq}} = F/\lambda$ expected if F were held fixed. In this only-fast-mode regime, the thermal disequilibrium,

$$\frac{T}{T_{\text{eq}}} = \frac{\lambda}{\lambda + \lambda' + \gamma} \quad (17)$$

is also a constant. Since the fast-mode dominates at present and at year 70 in a 1%/yr scenario, we would expect present-day T/T_{eq} and TCR/ECS to be approximately equal to this constant. Appendix Figure 3.6 illustrates that this approximation holds to within a few percent error. This exceptional accuracy is enabled by a cancellation at second-order between the surface-ocean inertia and the small, but non-zero, deep-ocean temperature. To see this, note that assuming quasi-equilibrium with a preindustrial deep ocean is equivalent to taking $\delta = C_s dT/dt - (\lambda' + \gamma)T_d \rightarrow 0$ in the surface ocean heat budget (Eqn. 10). Appendix Figure 3.6 demonstrates that $\delta \approx 0$, constraining T/T_{eq} to equal TCR/ECS at present.

In future scenarios illustrated by the RCPs, T/T_{eq} slowly increases over time due to the gradually evolving deep-ocean mode (Appendix Figure 3.6); the largest effect occurs for RCP-2.6, in which the surface temperature stabilizes, affording the deep-ocean the opportunity to raise T/T_{eq} from 0.61 at present to 0.74 by the end of the century.

A caveat to this analysis is the reliance on the idealized two-box model. A 1D diffusive model provides equivalently accurate fits to comprehensive models (Caldeira and Myhrvold, 2013), and would exhibit more intermediate degrees of equilibration and scenario-dependence. Because historical forcing has increased slower than the 1%/yr trajectory, a diffusive model would suggest that we are closer to thermal equilibrium with our current forcing than the 1%/yr-driven ocean is with its respective forcing at CO_2 doubling (year 70). Therefore, in a diffusive model, present-day $T/T_{\text{eq}} > \text{TCR/ECS}$ implies *even greater cooling after zero emissions*, as the closer the ocean is to thermal equilibrium with present-day forcing, the more the decline in forcing dominates the zero-emissions response.

ZECMIP computations

To account for natural variability, the zero-emissions temperature T_{ze} is taken to be the 20-year average centered on the time of zero emissions. The final temperature T_f is the average over the last 20 years in each simulation. The majority of ZECMIP simulations are of too-short duration to reach the final carbon and thermal equilibria. If the simulated final state is out of thermal equilibrium, planetary energy balance yields $\epsilon N_f = F_f - \lambda T_f$, where N_f is the top of the atmosphere energy imbalance (units of W/m^2), and here ϵ is the efficacy of planetary heat uptake. This serves as an alternative parameterization of the pattern effect (rather than λ' as in Eqn. 10), which we employ here because values of ϵ have been previously fit to the ZECMIP models (MacDougall et al., 2020). We approximate ϵ as constant and use the value fit to years 10 through 140 of the 1%/yr CO_2 computed in MacDougall et al. (2020).

Repeating the derivation of the ZEC formula with $T_f = (F_f - \epsilon N_f)/\lambda$ yields

$$\frac{\text{ZEC}}{T_{ze}} \approx \frac{(F_f - \epsilon N_f)/F_{ze}}{\text{TCR}/\text{ECS}} - 1, \quad (18)$$

which is plotted in Figure 3.2. A subset of models have a non-zero N at equilibrium with preindustrial forcing due to energy conservation error. To account for this, we let N_f be the difference between the energy imbalance in the final and preindustrial states. The preindustrial N is estimated as the y-intercept of a linear regression of N versus F over the first 20 years of data (see Appendix Figure 3.10).

The ZECMIP emissions are diagnosed by reconstructing the annual carbon budget,

$$\frac{\Delta \text{CO}_2}{\Delta t} = E - U, \quad (19)$$

where E is the emissions in GtC/yr, U is the combined land and ocean uptake in GtC/yr for each model, and CO_2 is the atmospheric mass of carbon dioxide in GtC. The atmospheric tendency is equal to $\log(1.01) \text{ CO}_2/\text{yr}$, and the annual time series of CO_2 and U are reported for each model, enabling the determination of the annual emissions. The cumulative airborne fractions are defined as $A(t) = \text{CO}_2(t)/C_{\text{emit}}(t)$ and are shown in Appendix Figure 3.11.

As shown in Fig. 3.1, all of the multi-century model results except for Bern-ecs5k and UVic demonstrate multi-century cooling in the idealized 1%/yr experiment. In light of Equation 3.4, the cause of UVic’s warming is a weak post-emissions uptake of carbon, which results in an anomalous $A_f \approx 0.26$, the highest in the ensemble. In Matthews and Zickfeld (2012), UVic simulations yield the finding that all-species ZEC is positive on multi-decadal timescales, contrary to other studies using FaIR (Smith et al., 2019; Dvorak et al., 2022). The integrated assessment here provides the larger context for this disagreement, and tip the scales in favor of FaIR’s cooling trend, as UVic’s carbon evolution is exceptional compared to the rest of the EMIC ensemble (see Appendix Figure 3.11). The carbon uptake in Bern-ecs5k is unremarkable, but the model has $\text{TCR}/\text{ECS} \approx 0.41$, the smallest in the EMIC ensemble and far below the AR6 estimate (0.58 [0.43 – 0.76]), yielding warming that therefore also appears unlikely.

Estimating the final airborne fraction

For the multi-century post-emissions response, carbon uptake after emissions is dominated by the oceanic carbon sink and constrained by carbonate chemistry. The second-order contribution from long-term land uptake is less constrained, but is on average ~ 100 GtC and still declining a millennium after emissions of 750 GtC end in ZECMIP models (see Appendix Figure 3.11). Here we follow Goodwin et al. (2007b) and estimate the final airborne fraction assuming no land uptake. The equilibrium perturbations to atmospheric and oceanic carbon are determined by the Revelle buffer factor (Revelle and Suess, 1957),

$$R = \frac{\delta p\text{CO}_2/p\text{CO}_2}{\delta[\text{DIC}]/[\text{DIC}]}, \quad (20)$$

where $[\text{DIC}]$ is the average concentration of dissolved inorganic carbon in the ocean. The DIC concentration of an oceanic parcel can be separated into a saturated and an anomaly component,

$$[\text{DIC}] = [\text{DIC}_{\text{sat}}] + [\text{DIC}'], \quad (21)$$

where the saturated component corresponds to the hypothetical concentration if brought into chemical equilibrium with the overlying atmosphere. Departures from saturation of roughly 10% are due to biological processes, air-sea exchange timescales, and ocean transport across temperature and solubility gradients (Williams and Follows, 2011). It is commonly assumed that the anomaly component is constrained by biological limitations (light and nutrients) and the ocean circulation, such that anthropogenic carbon is unlikely to substantially modify the anomaly component (Goodwin et al., 2007b; Williams and Follows, 2011; Williams et al., 2016). In this case, an infinitesimal emission δC_{emit} will be partitioned between the atmosphere and the total saturated DIC in the ocean according to the ratio $C_{\text{atm}}/(C_{\text{atm}} + \text{DIC}_{\text{sat}}/R)$, where $C_{\text{atm}} + \text{DIC}_{\text{sat}}/R$ is known as the *buffered carbon inventory*, C_{atm} is the mass of atmospheric carbon, and $\text{DIC}_{\text{sat}} \approx 35,000$ GtC is the mass of the saturated carbon inventory (Williams and Follows, 2011). For finite emissions, integrating

$$\frac{dC_{\text{atm}}}{dC_{\text{emit}}} = \frac{C_{\text{atm}}}{C_{\text{atm}} + \text{DIC}_{\text{sat}}/R}, \quad (22)$$

assuming a constant buffered carbon inventory (rather than constant R) results in a final equilibrium atmospheric carbon mass of

$$C_{\text{f}} \approx C_{\text{pre}} \exp \left(\frac{C_{\text{emit}}}{C_{\text{pre}} + \text{DIC}_{\text{sat}}/R} \right) \quad (23)$$

accurate to within 5% for cumulative emissions beneath 5000 GtC (Goodwin et al., 2007b) for an ocean at fixed temperatures. Oceanic uptake decreases with temperature, yielding higher atmospheric CO_2 , but this increase is ~ 10 ppm/K at equilibrium (Archer et al., 2004). The pre-industrial Revelle factor and buffered carbon inventory are estimated to be $R = 12.6 \pm .1$ and 3390 ± 50 GtC, respectively, given the pre-industrial ocean's average temperature, salinity, alkalinity, and $[\text{DIC}_{\text{sat}}]$ (Williams et al., 2016). The final airborne fraction $A_{\text{f}} = C_{\text{f}}/C_{\text{emit}}$ assuming $C_{\text{pre}} = 594$ GtC (280 ppm) and historical $C_{\text{hist}} = 680 \pm 65$ GtC is then $A_{\text{f}} = 0.19 [0.19 - 0.20]$ in this idealized ocean-atmosphere model. Substitution of C_{f} into the logarithmic forcing expression in Equation 3.8 yields a final temperature constraint (Goodwin et al., 2015),

$$T_{\text{f}} = \frac{C_{\text{emit}} \text{ ECS}}{\log(2) (C_{\text{pre}} + \text{DIC}_{\text{sat}}/R)}, \quad (24)$$

which is plotted (dashed blue line) in Figure 3.4.

FaIR model simulations

We run version 2.1 of the Finite amplitude Impulse Response model (FaIR), consisting of a three-layer energy balance coupled to a basic carbon-cycle model (Leach et al., 2021a). Carbon uptake is parameterized via a four-timescale impulse response function with state-dependent adjustments to incorporate carbon-climate feedbacks. Simple treatments of methane chemistry, aerosol direct and cloud-interaction effects, and other minor GHGs are also included (see Leach et al. (2021a) for details). We take advantage of the AR6-calibrated FaIR ensemble (Smith, 2024). This 841-member posterior ensemble was constrained out of a 1.5×10^6 -member prior ensemble to reproduce historical observations and the assessed AR6 ranges of key climate metrics. A full description of the calibration process can be found in ref (Smith, 2024).

For the historical ZEC experiment, we drive the FaIR ensemble with the Indicators of Global Climate Change (IGCC) emissions reconstruction (Forster et al., 2023). This reconstruction relies on up-to-date data sources to infer emissions from 1750 to 2022 following the AR6 methodology. To extend to 2023, we use the SSP2-4.5 projection (Meinshausen et al., 2020) harmonized to 2022 (Smith, 2024). In the historical ZEC experiment, we “zero emissions” in 2023 by instantaneously reverting all emissions to pre-industrial baselines. For the other SSP experiments, we follow the same procedure but zero emissions at the end-of-century. For the SSP1-2.6 and SSP4-3.4 pathways, peak temperatures occur within a decade of CO₂ net-zero, which is achieved roughly mid-century, and followed by a period of weakly negative CO₂ emissions. The minor decrease in cumulative emissions due to these negative emissions yields the slight x-offsets between the equilibrium cumulative emissions (circles) and cumulative emissions at peak warming (squares) in Figure 4a. For stronger negative emissions that restore atmospheric CO₂ to the pre-industrial state over centennial timescales, the temperature trajectory is expected to decline towards the origin in Figure 3.4a along a curve intermediate between the equilibrium and peak lines (Koven et al., 2023, Figure 2b). Finally, note that the FaIR simulations assume land-use emissions cease, yet land-use activity continues such that the forcing of -0.2 W/m^2 due to land-use albedo change persists over the next millennium. If land-use activity instead ceased, allowing for ecosystem restoration, additional carbon uptake would lower the final CO₂ forcing realized in the FaIR simulations. For instance, complete uptake of historical land-use emissions yields a change in final CO₂ forcing of -0.35 W/m^2 in the historical case.

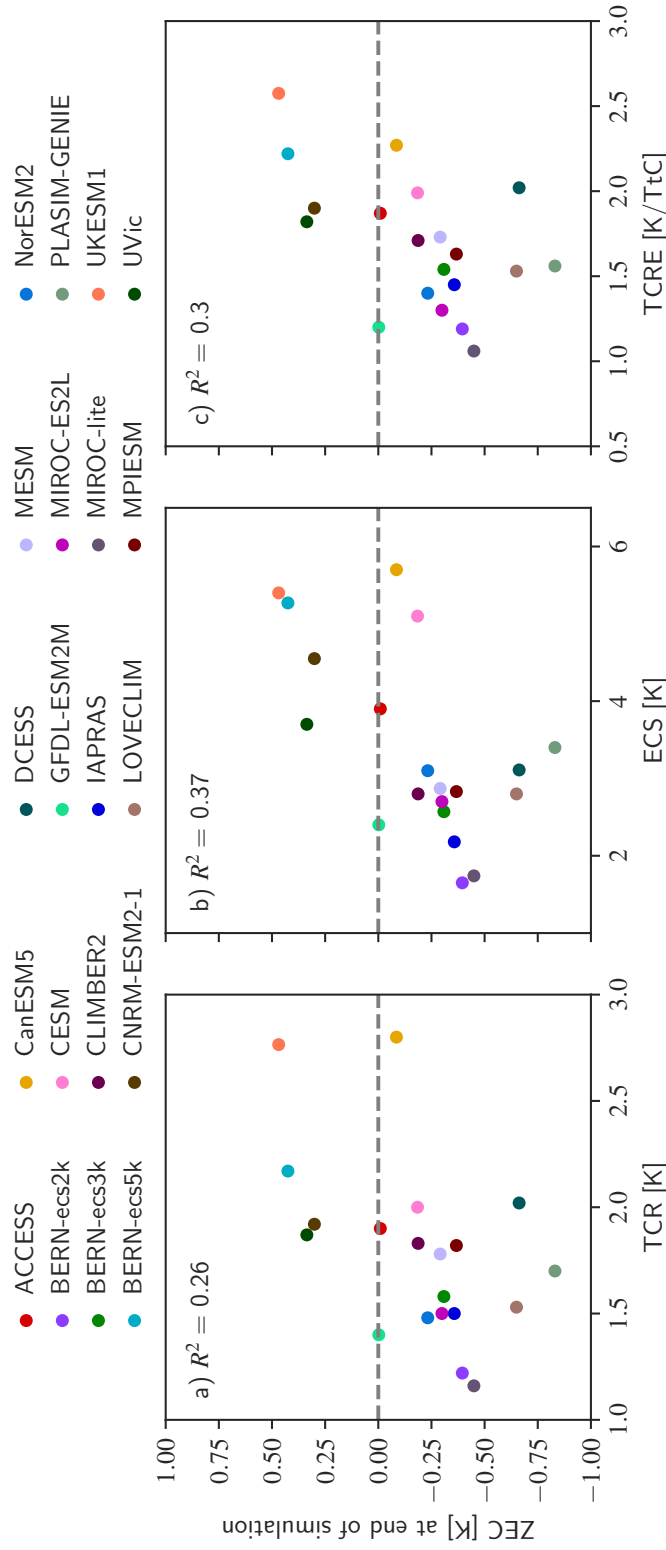


Figure 3.5: ZEC correlates weakly with established metrics. Correlations between ZEC realized in the ZECMIP 1000 GtC simulations (of varying duration) shown in Figure 1 of the main text and the transient climate response (left), equilibrium climate sensitivity (center), and the transient climate response to cumulative emissions (right).

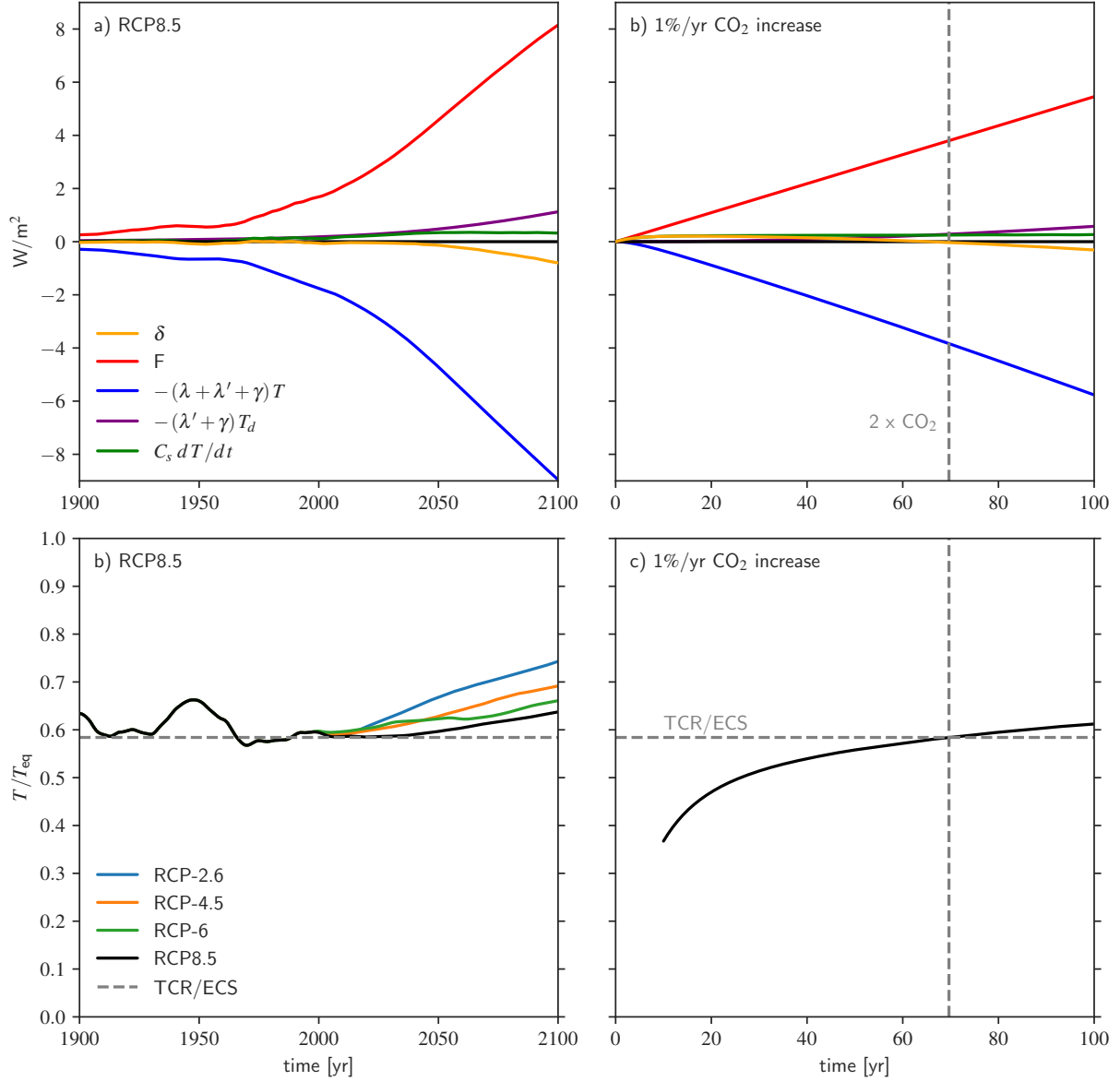


Figure 3.6: TCR/ECS constrains thermal disequilibria. Top row: contributions to the mixed-layer heat budget of the energy-balance model with parameters fit to the CMIP5 ensemble average (Geoffroy et al., 2013) for RCP8.5 (left) and a 1%/yr increase in CO_2 (right). Here $\delta = c_s dT/dt - (\lambda' + \gamma)T_d$ captures the departure from constant T/F and thus constant T/T_{eq} . Bottom row: ratio of the transient surface temperature (T) to the equilibrium temperature if the forcing were held fixed (T_{eq}) in various RCPs (left) and the 1%/yr CO_2 increase (right).

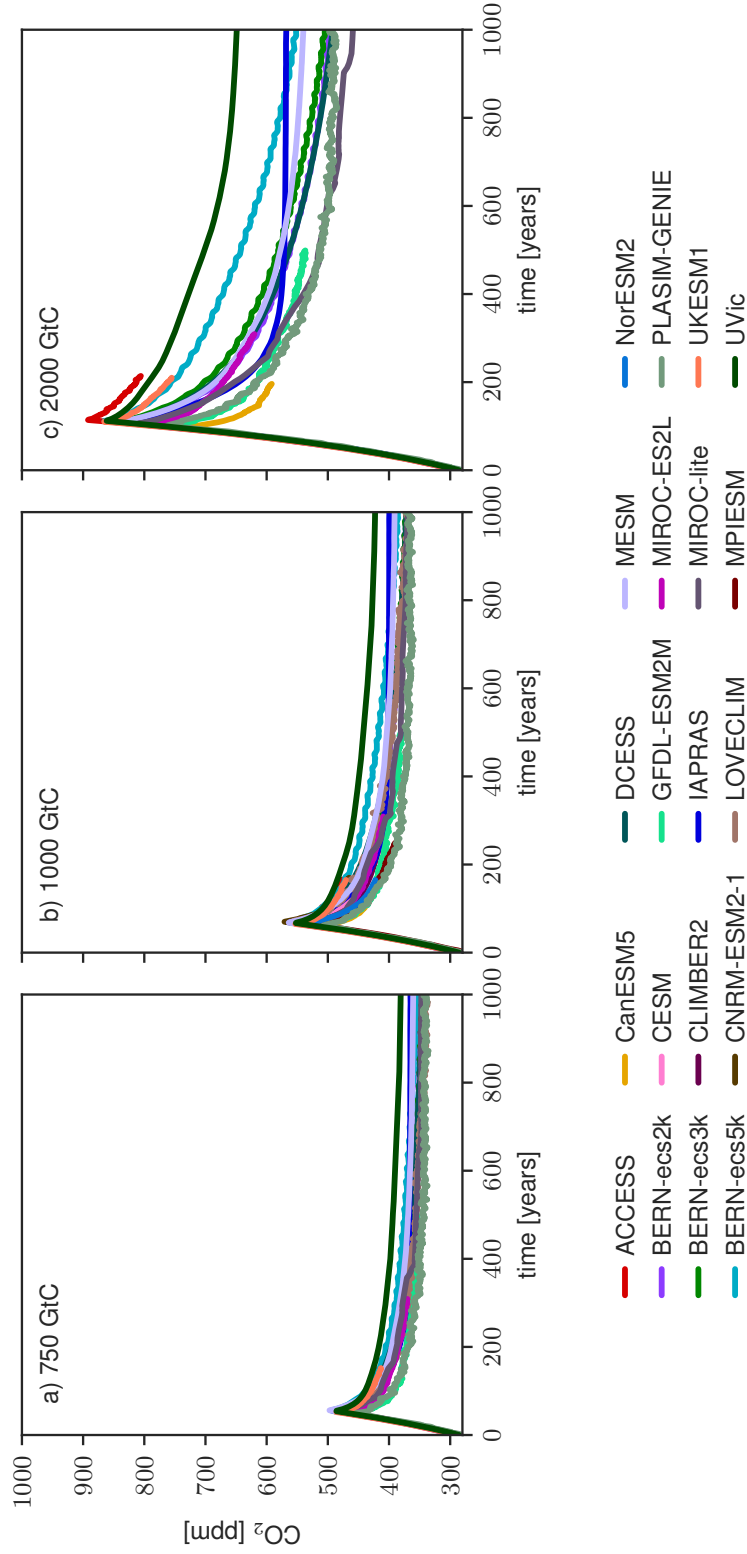


Figure 3.7: Post-emissions CO₂ evolution. Atmospheric concentration of carbon dioxide in ZECMIP models for emissions pathways that yield a 1%/yr. growth in atmospheric carbon dioxide and cease at cumulative emissions of 750 GtC (left), 1000 GtC (center), and 2000 GtC (right).

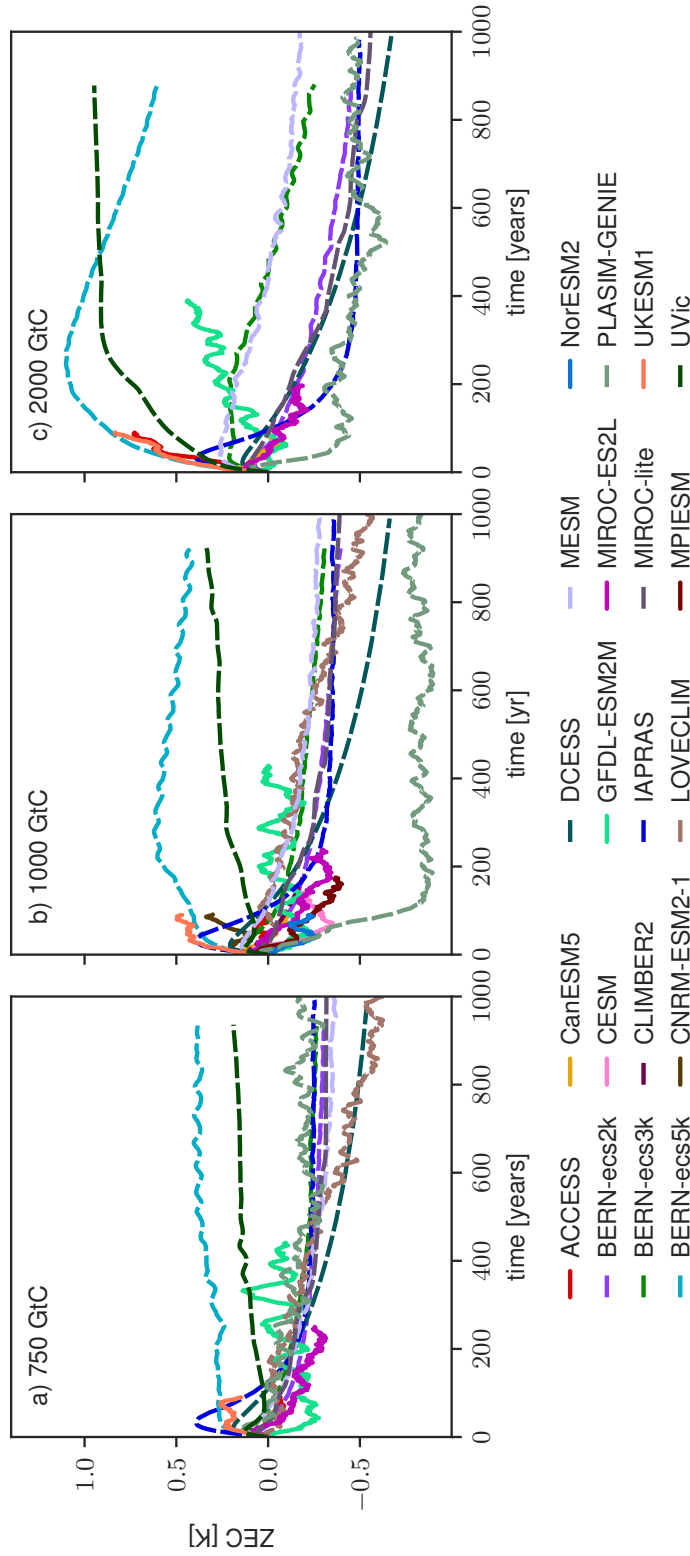


Figure 3.8: Post-emissions temperature evolution. Temperature change after emissions cessation as realized by full complexity (solid) and intermediate complexity (dashed) Earth system models for emissions pathways that yield a 1%/yr. growth in atmospheric carbon dioxide and cease at cumulative emissions of 750 GtC (left), 1000 GtC (center), and 2000 GtC (right).

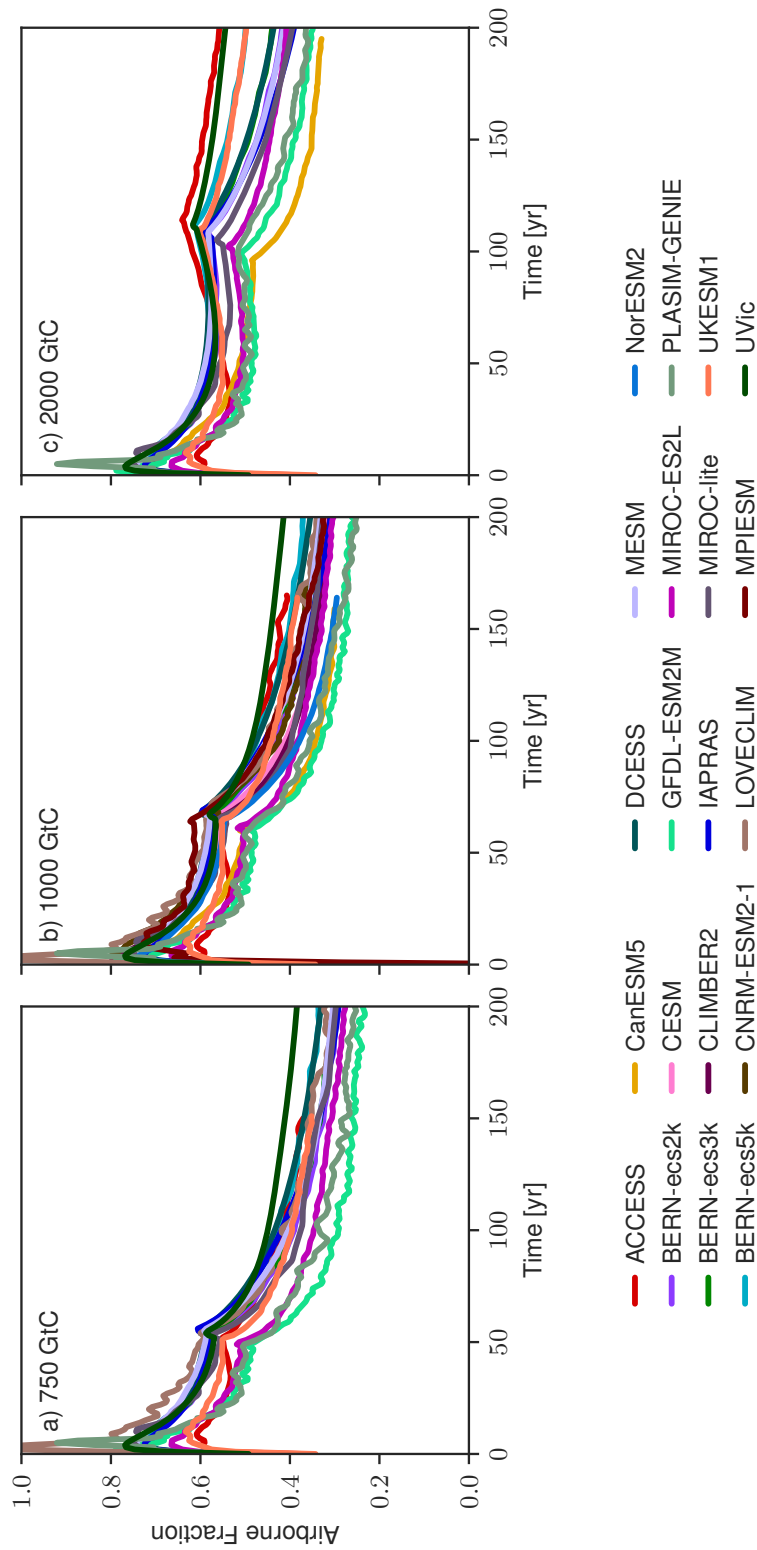


Figure 3.9: Airborne fraction reconstructed from carbon budget. The fraction of cumulative emissions present in the atmosphere for the ZECMIP 1%/yr experiments with cumulative emissions of 750 GtC (left), 1000 GtC (center), and 2000 GtC (right). Emissions are diagnosed by closing the global carbon budget.

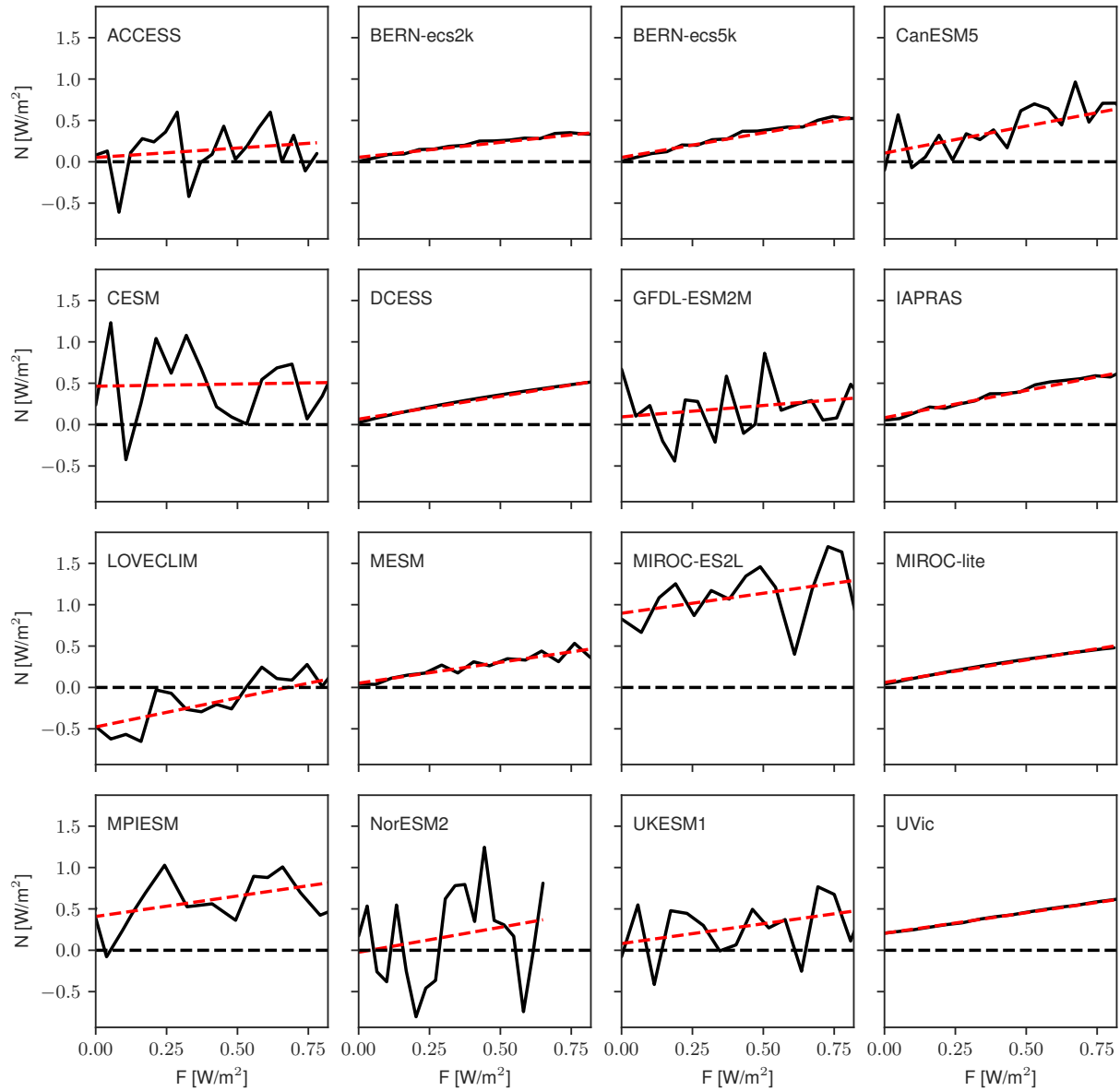


Figure 3.10: Not all models conserve energy for pre-industrial. The top-of-the-atmosphere energy imbalance N versus the radiative forcing for ZECMIP models over the first 20 years of the 1%/yr CO_2 experiments. Linear regression (red dotted) is performed to estimate the pre-industrial N , which may be non-zero due to energy conservation error.

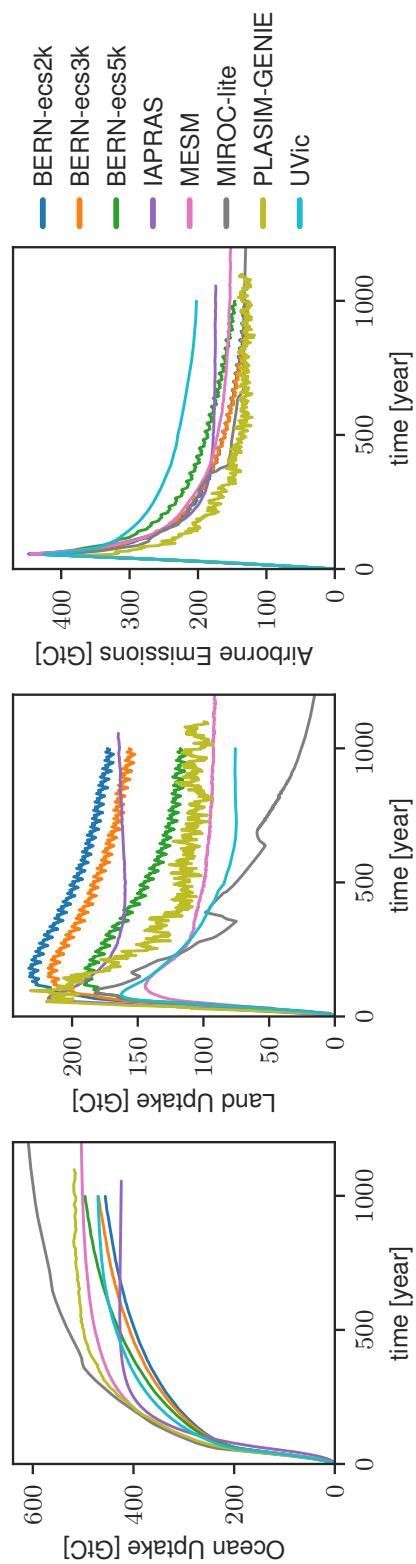


Figure 3.11: Millennial-scale carbon uptake. Carbon uptake of anthropogenic emissions by the ocean (left) and land (center) compared to airborne emissions (right) in the 750 GtC 1%/yr CO₂ experimental results for long-running EMICs.

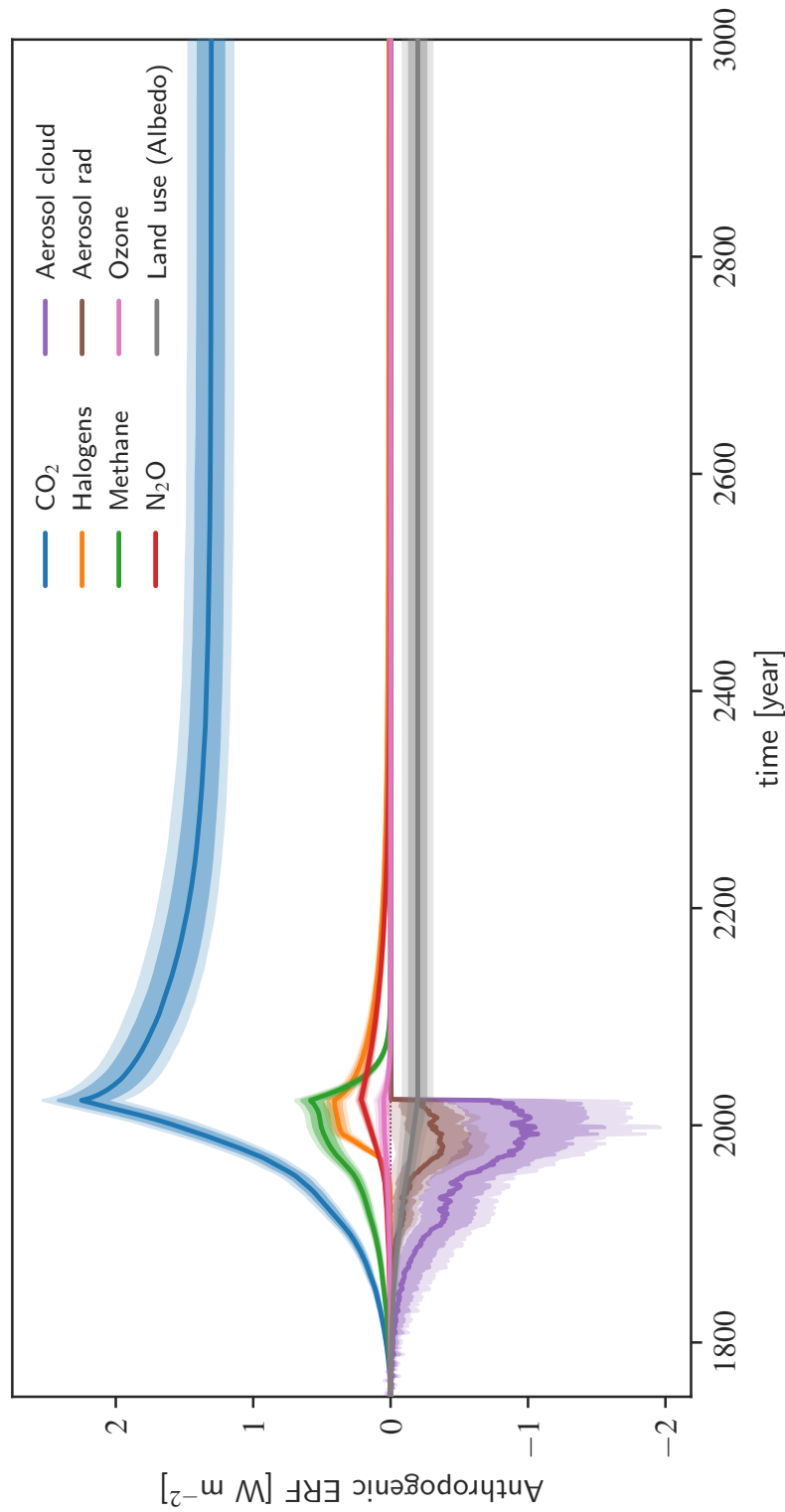


Figure 3.12: Anthropogenic ERF if emissions cease. ERF computed in FaIR ensemble (Thornhill et al., 2021b) due to historical emissions followed by cessation of all species. Dark shading shows the 16th-84th interquartile range, and light shading shows the 5th-95th.

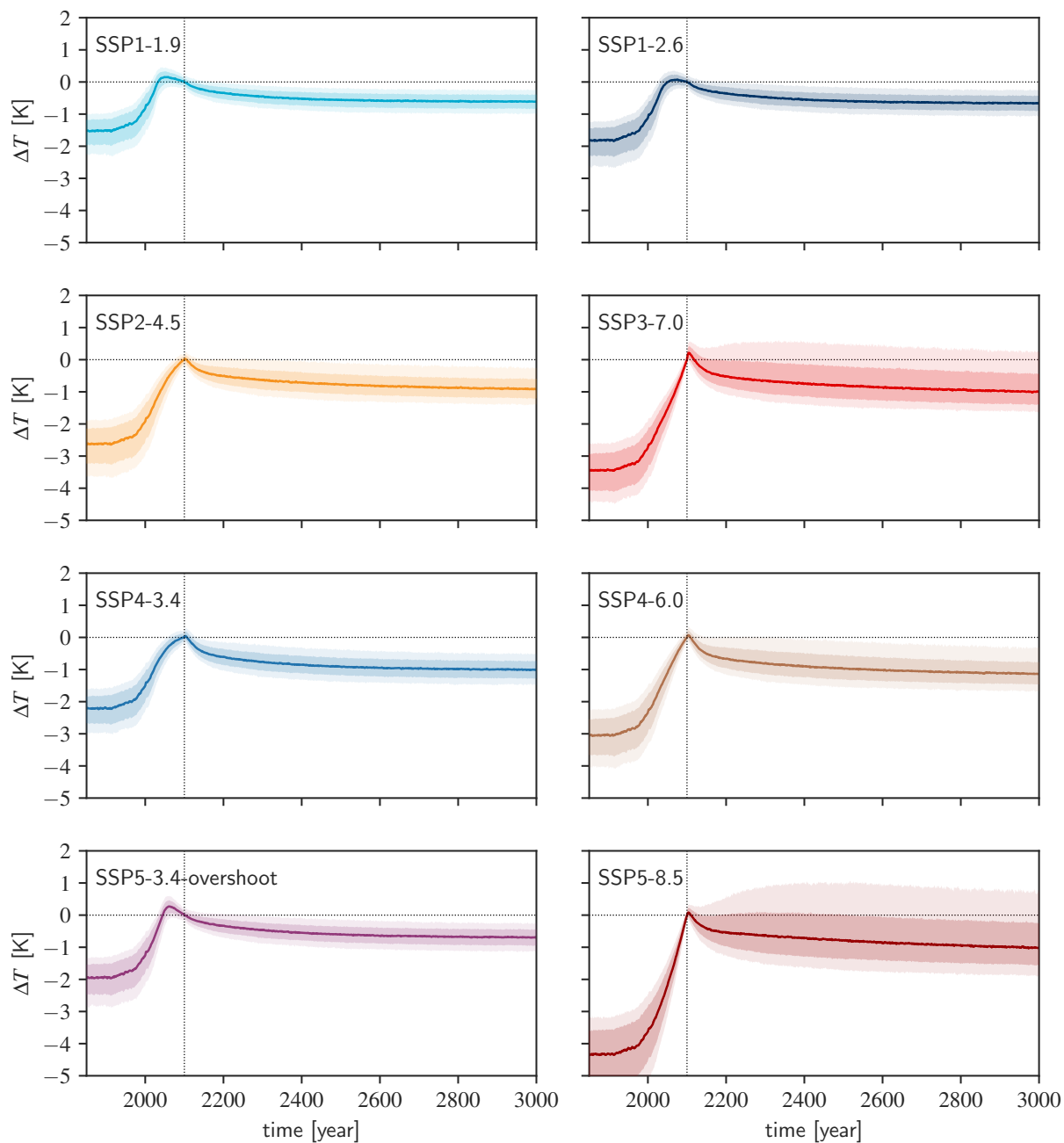


Figure 3.13: Similar ZEC across the SSPs. Millennial evolution of the temperature anomaly relative to emissions cessation at 2100 in the Shared Socioeconomic Pathways. Dark shading shows the 16th-84th interquartile range, and light shading shows the 5th-95th.

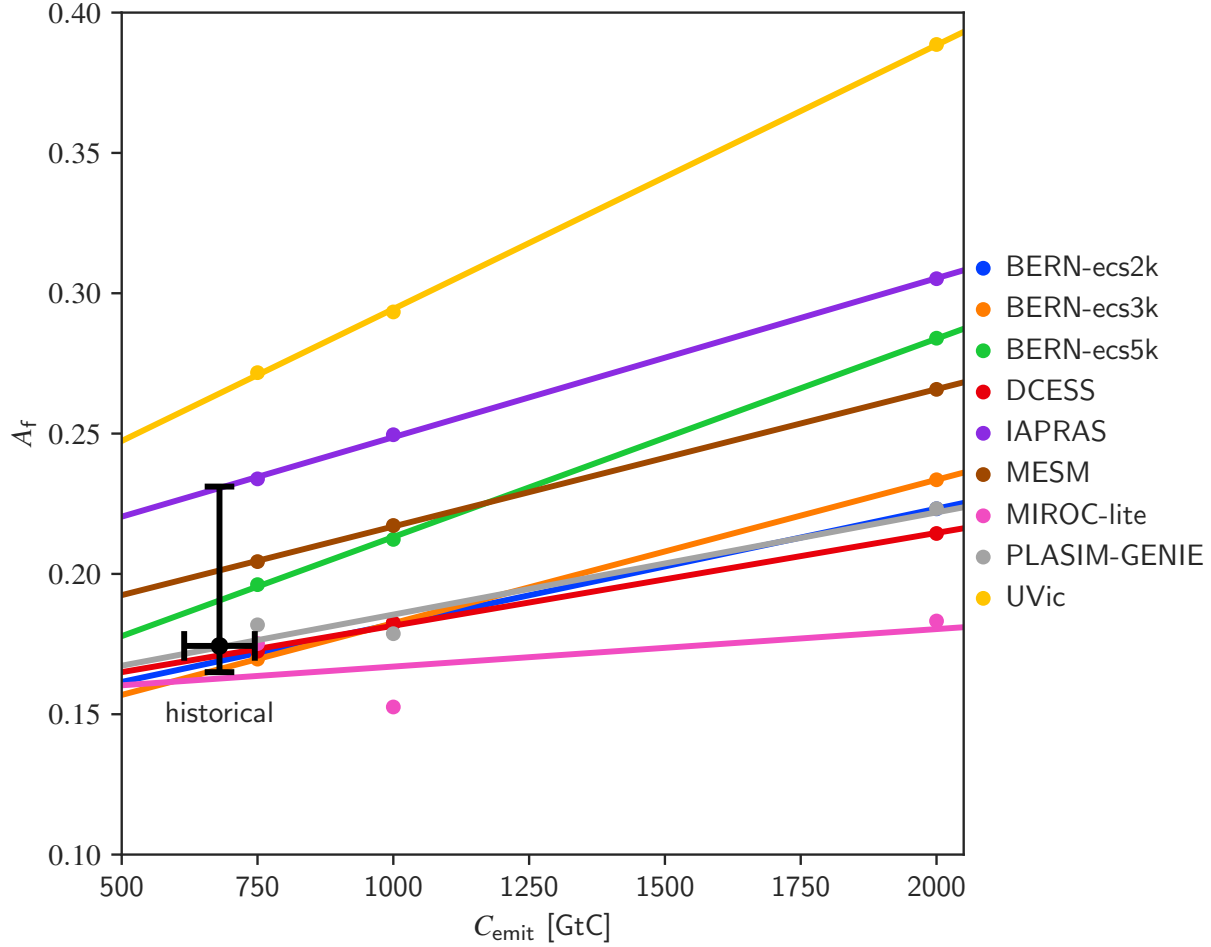


Figure 3.14: Historical estimate of A_f inferred from ZECMIP experiments. To infer A_f for historical emissions of $680 (\pm 65)$ GtC (Friedlingstein et al., 2022) from the ZECMIP data, we Monte Carlo sample the models, and extrapolate from the per-model linear fits to the variation of A_f with cumulative emissions plotted above, given the millennial ZECMIP experiments with cumulative emissions of 750, 1000, and 2000 GtC, respectively (colored markers). This yields the median historical estimate (black marker) and associated 68% confidence interval.

Chapter 4

A closure for the virtual origin of plumes

An isolated source of surface buoyancy, be it a campfire or burning city, gives rise to a turbulent plume. Well above the surface, the plume properties asymptote to the well-known solutions of Morton, Taylor, and Turner (MTT), but a closure is still lacking for the virtual origin. A closure for the virtual origin is sought here in the case of a turbulent plume sustained by a circular source of surface buoyancy in an unstratified and unsheared fluid. In the high Reynolds number limit, it is argued that all such plumes asymptote to a single solution. Direct numerical simulation (DNS) of this solution exhibits a virtual origin located a distance below the surface equal to 1.1 times the radius of the buoyancy source. This solution is compared to the previously used assumption that the MTT plume is fully spun-up at the surface, and that assumption is found to give buoyancies that are off by an order of magnitude. With regards to the citywide firestorm triggered by the nuclear attack on Hiroshima, it is found that the spun-up-at-surface MTT solution would have trapped radioactive soot within about a hundred meters of the surface, whereas the DNS solution presented here corroborates observations of the plume reaching well into the troposphere.

4.1 Introduction

In the presence of gravity, a steady isolated heating at the bottom of an unstratified fluid will produce a plume: a buoyant column of rising fluid whose steady shape is maintained (in spite of its ascent) by the entrainment of surrounding fluid. A pleasant and familiar example is the turbulent and smoky plume generated by a campfire. A decidedly less pleasant example is the plume generated by a city-sized fire ignited by a nuclear weapon. This latter case,

The content of this chapter was previously published in Tarshish and Romps (2021).

with its attendant threat of stratospheric soot injection and climate perturbation, motivates the objective of this paper: to characterize the near-surface spin-up of such plumes.

For the sake of simplicity, we focus here on buoyancy-driven plumes in an unstratified fluid initially at rest, or in cases where the background flow is small enough to have a negligible effect on the plume’s gross characteristics. Aside from the surface fire, we assume there are no diabatic sources of heat. In the atmosphere, this means that the air is taken to be dry, i.e., we focus on the dynamics of plumes below their lifting condensation level.

Morton et al. (1956) (hereafter, MTT) established the foundational closure for plumes far from their source of buoyancy in a neutral Boussinesq fluid: by dimensional analysis, MTT postulated that the plume’s turbulent mixing length is proportional to the plume’s width. As reviewed in Turner (1986), numerous experiments in the laboratory (Carazzo et al., 2006), flows in nature (Woods, 2010), and numerical simulations (Devenish et al., 2010) have tested the MTT theory and found close agreement. Subsequent work has built on this foundation, analyzing plumes in more complex settings, such as non-Boussinesq fluids (Rooney and Linden, 1996) and stratified environments (Woods, 2010).

For many applications, motion far above the source is of primary interest (e.g., Carazzo et al., 2006, Table 1). Near the source, however, the flow is manifestly different from the canonical MTT plume: a horizontal circulation converges fluid over the source, mixes in environmental air, and launches the fluid upward. If we wish to predict the fate of the buoyant fluid (e.g., its eventual level of neutral buoyancy), we must be able to predict the height at which the plume has largely transitioned to an MTT regime, as well as the plume’s buoyancy, width, and vertical velocity at that height.

At present, there is no rigorous theory for the near-surface dynamics of buoyancy-driven plumes. Indeed, our search of the literature turned up neither empirical data on nor well-resolved simulations of the near-surface flow in response to an isolated heating at a no-slip lower boundary. Absent the requisite data or theory for the spin-up layer, some researchers have resorted to the ad hoc practice of initializing the MTT plume as fully spun-up at the surface with the full radius of the source (Freitas et al., 2007; Paugam et al., 2016) while others have neglected the source’s radius entirely (Penner et al., 1986; Carrier et al., 1985; Manins, 1985), modeling the plume’s source as point-like at the surface. By neglecting the spin-up boundary layer entirely, both of these initializations have the non-physical consequence of $w \neq 0$ at the surface — a mismatch that may play a role in why wildfire plume predictions compare poorly to observations (Paugam et al., 2016).

In the MTT solution, the turbulent plume has the shape of a cone. At the vertex of the cone, the radius of the plume is zero and the buoyancy and vertical velocity are infinite. Of course, no plume in reality has such a vertex, but real plumes (in unstratified, unsheared fluids) do asymptote to the MTT solution sufficiently far above their heating source. At those heights, the plume is fully characterized by only two quantities: the buoyancy source rate and the location of the vertex of the best-fit cone, which is called the virtual origin. The two ad hoc initializations described above differ only in their choice of virtual origin. When initializing a plume as emanating from a point at the surface, the height of the virtual origin

z_v is zero. For the spun-up-at-surface plume, whose width at the surface equals the width of the surface buoyancy source, $z_v < 0$. In either case, however, the only other parameter – the buoyancy source rate – is set to the actual rate. Therefore, to model real plumes far above the surface, what is needed is a closure for the virtual origin.

Although a great number of papers have been written about plumes, we were unable to find any closure in the literature for the virtual origin of a plume with the following three properties relevant to real-world fires: a plume that has 1. well-resolved turbulence and that is 2. sourced only by buoyancy at a 3. no-slip surface. To fill that gap, we present here a closure for the virtual origin of such plumes using direct numerical simulations in two and three dimensions. To illustrate the importance of using the correct virtual origin, we will estimate the level of neutral buoyancy of the Hiroshima nuclear firestorm plume using the new closure and, for comparison, the commonly used spun-up-at-surface closure. Before we begin, however, let us briefly review some of the prior literature to see the ways in which conditions 1, 2, and 3 have been missing from previous studies.

Two theoretical predictions have been made for the virtual origin of a plume driven by a circular source of buoyancy. Those calculations used turbulence closures to derive analytic solutions for a buoyancy-driven plume that is initialized by a finite-area circular source rather than a point source (as is the case for the MTT plume). The turbulence closure adopted by Hunt and Kaye (2001) modeled the plume’s fractional entrainment rate as α/r , where r is the plume’s radius and α is the same dimensionless constant found empirically for far-field plumes (see the discussion of α in section 2). That closure leads to an analytic solution with a virtual origin at the same height as the circular source of buoyancy. Ciriello and Hunt (2020) replaced α with a linear function of the plume’s Richardson number and found an analytic solution (their $\Gamma_0 \rightarrow \infty$ solution) in which the virtual origin is located below the physical source of buoyancy by a distance of about 3.34 times the physical source’s radius. The accuracy of these predictions is uncertain given their dependence on simplified approximations of the plume dynamics.

Three types of empirical approaches have been dominant in plume studies: water-tank experiments, heated-air experiments, and numerical simulations. In the water-tank experiments (Colomer et al., 1999; Friedl et al., 1999; Epstein and Burelbach, 2001; Kaye and Hunt, 2009), it has been common to pump salty water into a fresh water tank, initializing the near-source fluid with both mass and momentum in addition to buoyancy, thereby violating condition 2. Compounding the problem, few eddies were present in the above experimental near-source flows (see Figures 1 and 2 of Kaye and Hunt, 2009) suggesting laminar growth (Friedl et al., 1999). Therefore, these water-tank experiments do not satisfy conditions 1 and 2.

In the heated-air experiments, the plumes are typically generated by a heated metal disk. However, the majority of these studies placed the disk atop a cylinder (e.g., Bouzinaoui et al., 2007; Fontaine et al., 2006; Vuong Pham et al., 2005; Popiolek et al., 1998; Elicer-Cortés, 1998; Kofoed and Nielsen, 1990), producing a free-slip flow rather than condition 3 of a no-slip surface. Kuznetsov et al. (2019) and Mahmoud et al. (2009) enforced the no-slip

boundary condition by heating metal disks embedded in a lower surface, but these studies did not compute the virtual origin nor present data from which the virtual origin can be inferred.

Numerical simulations have also investigated plumes evolving from a circular heat source (Devenish et al., 2010; Fontaine et al., 2006; Plourde et al., 2008). The large-eddy simulations of Devenish et al. (2010) used a horizontal resolution that was appropriate for the far-field dynamics, but coarse compared to the source’s width, and so did not meet condition 1 of having well-resolved turbulence. The simulated plume of Fontaine et al. (2006) had a free-slip elevated source and so did not meet condition 3. Finally, Plourde et al. (2008) performed higher-resolution simulations of a plume fed by a circular source of buoyancy at a no-slip surface, but found an “unrealistically laminar” plume, which was then made to be approximately turbulent by continuously adding white noise to the near-surface vertical velocity. This suggests that, despite the high reported Reynolds number of 3055, this simulation did not satisfy condition 1 of having well-resolved turbulence. Furthermore, Plourde et al. (2008) reported that the “disturbance added to the flow close to the heat source is an important parameter that controls the onset and location of transition [from a laminar to a turbulent state]”. In section 4.3, we argue that the resolution in Plourde et al. (2008) was not sufficient to directly resolve dissipation at the Kolmogorov microscale, which is required for a direct numerical simulation to perform properly.

4.2 Theory

In this section, we will review the Morten-Taylor-Turner (MTT) solutions, the sensitivity of those solutions to the virtual origin, and the high-Reynolds-number limit.

Review of MTT

Our results build on the foundational theory of convective plumes put forth by MTT, which we now review. MTT formulates bulk conservation equations for a plume’s volume, momentum, and buoyancy fluxes in an unstratified Boussinesq fluid. In doing so, the environment is taken to have zero vertical momentum, and buoyancy is assumed to be the dominant force. In a neutrally buoyant fluid, the MTT equations read

$$\frac{d}{dz} (r^2 w) = 2r u_e, \quad (4.1)$$

$$\frac{d}{dz} (r^2 w^2) = r^2 b, \quad (4.2)$$

$$r^2 w b = Q = \text{constant}, \quad (4.3)$$

where the plume is described by radius r , buoyancy b , and vertical velocity w . By convention, Q is the buoyancy source divided by a factor of π . Turbulent entrainment of environmental air into the plume is quantified by the entrainment velocity u_e .

The MTT turbulence closure is $u_e = \alpha w$ for some dimensionless constant α . This closure stems from the reasonable guess that the sizes and speeds of the eddies responsible for entrainment scale with the size and speed, respectively, of the plume itself. Given this closure, the equations admit an analytical solution for the case in which the buoyancy source is located at a point,

$$r = \frac{6\alpha}{5}(z - z_v), \quad (4.4)$$

$$w = \frac{5^{1/3}}{2\alpha^{1/3}} Q^{1/3} r^{-1/3} \quad (4.5)$$

$$b = \frac{2\alpha^{1/3}}{5^{1/3}} Q^{2/3} r^{-5/3}. \quad (4.6)$$

In this solution, the buoyancy source is located at $r = 0$ and $z = z_v$. This point is referred to as the virtual origin. Away from the surface, real plumes asymptote to an MTT solution (with Q set by the physical buoyancy source) but with a virtual origin that is, in general, not located at the surface. Geometrically, z_v marks the apex of the cone swept out by the plume far above the surface. While fundamental to the solution, the MTT theory does not predict z_v and does not attempt to describe the near-surface dynamics. In this respect, real plumes remain an unsolved problem: a closure is needed for the virtual origin.

The virtual origin

In laboratory and numerical studies, the virtual origin is fit ex post facto to experiments (e.g., Carazzo et al., 2006, and references therein). Table 4.1 collects prior measurements. The relevance of these past experiments to atmospheric plumes, however, is unclear, as no past experiment has satisfied conditions 1, 2, and 3 described above. Lacking solid experimental footing, predictive studies on atmospheric plumes have relied on the ad hoc practice of ignoring the spin-up layer altogether. Some authors assume the plume is fully spun-up at $z = 0$ with radius equal to the radius R of the patch of surface heating (usually taken to be a circle): enforcing this spun-up-at-surface condition on $r(z)$ requires that the virtual origin be placed at $z_v = -\frac{5R}{6\alpha} \approx -9R$ (Paugam et al., 2016; Freitas et al., 2007), given the canonical $\alpha \approx .1$ (Carazzo et al., 2006). Other authors make the ad hoc choice of placing the virtual origin at the surface by setting $z_v = 0$ (Penner et al., 1986; Carrier et al., 1985).

To determine whether these choices of z_v are consequential, we note that $b(z, z_v) \propto (z - z_v)^{-5/3}$ and compute the ratio,

$$\frac{b(z, z_v = 0)}{b(z, z_v = -9R)} = \left(1 + \frac{9R}{z}\right)^{5/3}. \quad (4.7)$$

If we are interested in the plume's buoyancy at heights comparable to the horizontal extent of the buoyancy source, then we could plug $z = R$ into this expression. This yields a buoyancy

ratio of 46. Thus, the choice of initialization has immense consequences for the plume's height of neutral buoyancy and, in the case of an urban firestorm, how much climate-altering soot reaches the upper atmosphere. If the virtual origin is not particularly sensitive to relaxing conditions 1, 2, and 3, then prior experimental work on related plumes is relevant, and suggests $z_v = 0$ is a more plausible assumption in light of the data in Table 4.1. Given the array of varied and conflicting initializations, we seek to definitively establish the plume's virtual origin for the case of interest with direct numerical simulations.

The turbulent limit

The success and broad applicability of the MTT theory rests on the observation that fully developed plumes appear to exhibit a nearly universal rate of change in radius with respect to height, independent of the experimental setup or fluid composition. This growth rate, captured by the entrainment rate α , is found to be close to 0.1 in a wide range of experiments (as reviewed in Carazzo et al., 2006). Scorer (1957), who first noted this invariance, hypothesized the existence of a universal self-similar solution that all plumes converge toward after turbulence dominates the flow.

For a neutral Boussinesq fluid, the flow is governed by the non-dimensional Reynolds number Re and the Prandtl number Pr , and Scorer's hypothesis implies that the large-scale features of the plume should converge to a single asymptote as Re increases, independent of Pr . Indeed, turbulent experiments with differing Pr (e.g., plumes in water versus air) demonstrate similar entrainment rates (Carazzo et al., 2006).

In this study, we perform direct numerical simulations that allow us to explicitly set the non-dimensional parameters and explore this high- Re convergence. To fix terms and define the non-dimensional parameters, we study plumes that are solutions to the Boussinesq equations for an unstratified, incompressible fluid,

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla \left(\frac{p'}{\rho_0} \right) + b \mathbf{e}_z + \nu \nabla^2 \mathbf{u}, \quad (4.8)$$

$$\partial_t b + \mathbf{u} \cdot \nabla b = \kappa \nabla^2 b, \quad (4.9)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (4.10)$$

where $\mathbf{u} = (u, v, w)$ is the velocity, ν is the kinematic viscosity, and κ is the thermal diffusivity. With a constant buoyancy flux F within radius R , the boundary conditions are

$$\mathbf{u}|_{z=0} = 0, \quad \left. \frac{\partial b}{\partial z} \right|_{z=0} = \begin{cases} -F/\kappa & r \leq R \\ 0 & r > R \end{cases}, \quad (4.11)$$

where $r = \sqrt{x^2 + y^2}$. Casting the equations in non-dimensional form, we introduce dimensional scales $B = F^{2/3} R^{-1/3}$, $U = F^{1/3} R^{1/3}$, $T = F^{-1/3} R^{2/3}$, and $P = F^{2/3} R^{2/3}$. Denoting

	Type	Surface Fluxes	BC	Re	z_v [R]
Devenish et al. (2010)	LES	Buoyancy	no slip	N/A	-1.4
Fontaine et al. (2006)	LES	Buoyancy	free slip	20000	-2.2 or -1.2
Plourde et al. (2008)	DNS	Buoyancy, Momentum Noise	no slip	N/A	-2.2
Bouzinaoui et al. (2007)	Heated air	Buoyancy	free slip	20000	-2
Kofoed and Nielsen (1990)	Heated air	Buoyancy	free slip	100	-.88
Elicer-Cortés (1998)	Heated air	Buoyancy	free slip	800	-.86
Kaye and Hunt (2009)	Salt water	Buoyancy, Volume, Momentum	free slip	N/A	-2
Colomer et al. (1999)	Salt water	Buoyancy, Volume, Momentum	free slip	4000	-1.6
Ciriello and Hunt (2020)	Theory	Buoyancy-only limit	N/A	N/A	-3.34
Hunt and Kaye (2001)	Theory	Buoyancy-only limit	N/A	N/A	0.0

Table 4.1: Comparison of z_v across prior work. See text for discussion. Fontaine et al. (2006) reports -2.2R and -1.2R for domain ventilation velocities of .15 m/s and .02 m/s, respectively. Least-squares fit to data in Figure 1 of Plourde et al. (2008) yields the reported z_v . Since the simulation of Plourde et al. (2008) appears not to have been DNS, it is not possible to calculate its Reynolds number.

non-dimensional variables by a caret, we get

$$\partial_t \hat{\mathbf{u}} + (\hat{\mathbf{u}} \cdot \hat{\nabla}) \hat{\mathbf{u}} = -\hat{\nabla} \hat{p} + \hat{b} \mathbf{e}_z + \frac{1}{\text{Re}} \hat{\nabla}^2 \hat{\mathbf{u}}, \quad (4.12)$$

$$\partial_t \hat{b} + \hat{\mathbf{u}} \cdot \hat{\nabla} \hat{b} = \frac{1}{\text{Re Pr}} \hat{\nabla}^2 \hat{b}, \quad (4.13)$$

$$\hat{\mathbf{u}}|_{\hat{z}=0} = 0 \quad \frac{\partial \hat{b}}{\partial \hat{z}} \bigg|_{\hat{z}=0} = \begin{cases} -\text{Re Pr} & \hat{r} \leq 1 \\ 0 & \hat{r} > 1 \end{cases}, \quad (4.14)$$

where we have defined the Prandtl and Reynolds numbers as

$$\text{Pr} = \frac{\nu}{\kappa}, \quad \text{Re} = \frac{F^{1/3} R^{4/3}}{\nu}. \quad (4.15)$$

In this analysis, we take the atmospheric value of $\text{Pr} = .7$ and explore the solution's dependence on Re . Given the above non-dimensional equations, the non-dimensional solution is fully specified by the global Re and Pr . By dimensional reasoning, the plume's time-averaged center-line vertical velocity w_c and buoyancy b_c must take the forms $b_c = \hat{b}_c(\text{Re}, \text{Pr}, \hat{z}) F^{2/3} R^{-1/3}$ and $w_c = \hat{w}_c(\text{Re}, \text{Pr}, \hat{z}) F^{1/3} R^{1/3}$, where the non-dimensional scaling functions are at present unknown.

The global Re defined above is closely related to the plume's local Reynolds number Re_{local} expressed in terms of the plume's vertical velocity w_c as

$$\text{Re}_{\text{local}} = \frac{w_c r}{\nu} = \text{Re} \hat{w}_c \hat{r} = \frac{5^{1/3}}{2\alpha^{1/3}} \text{Re} \hat{r}^{2/3}, \quad (4.16)$$

where we have applied the MTT scalings (Eqn. 4.5) and emphasize that the global Re is a constant fixed by the source's scales and the fluid's viscosity. We see that the plume's local Reynolds number increases as the plume rises and widens. Growth of the MTT plume, however, only explicitly depends on α , and laboratory experiments peg α to a height-independent constant value (Carazzo et al., 2006). The fact that α remains fixed at the universal value of approximately 0.1 as Re_{local} increases is a consequence of the fluid flow having already asymptoted to the high- Re_{local} limit. In what follows, we will show that this same asymptotic behavior applies not just to the far-field MTT plume, but also to the near-surface spin-up layer that initializes the plume.

4.3 Model

We solve the non-dimensionalized Boussinesq equations (Eqns. 4.12, 4.13, and 4.14) with Dedalus, a pseudo-spectral framework for direct numerical simulations (Burns et al., 2020). The simulation is run with $\text{Pr} = .7$ and $\text{Re} = 10^3$ in a cubic domain with side length 12 (recall that the equations have been non-dimensionalized). Two-dimensional simulations are

run in an identical square domain at various Re . In the z -dimension, we express the fields as a Chebyshev series. At the surface, no-slip boundary conditions are enforced and a circular spectrally-smooth source of buoyancy is specified,

$$\left. \frac{\partial \hat{b}}{\partial \hat{z}} \right|_{\hat{z}=0} = -\frac{\text{RePr}}{2} \left[1 - \tanh \left(\frac{\sqrt{\hat{x}^2 + \hat{y}^2} - 1}{\Delta r} \right) \right], \quad (4.17)$$

with $\Delta r = .01$. In the horizontal, the fields are expanded in Fourier series.

To be considered a direct numerical simulation, the spectral modes, when projected onto a collocation grid in physical space, must satisfy the requirement that the grid spacing Δx is $\mathcal{O}(\eta)$ where η is the Kolmogorov length (Moin and Mahesh, 1998). In turn, η may be solved for by noting that the length-scale of energy injection R is linked to the dissipation scale η via the Kolmogorov scaling $\eta/R = \text{Re}^{-3/4}$. Setting $\text{Re} = 10^3$ and expanding the fields horizontally in $N = 512$ Fourier modes in Dedalus yields a physical space collocation grid with uniform horizontal resolution $\Delta x = \Delta y = 12R/512 \approx 4\eta$.

For the vertical Chebyshev basis, the collocation grid has variable resolution with a higher density of grid points near the surface. Grid points satisfying $z < R$ have an average grid spacing of $\Delta z \approx 1.1\eta$. The resolution of the surface flow thus meets the $\mathcal{O}(\eta)$ criterion and is comparable to past spectral resolutions listed in Table 1 of Moin and Mahesh (1998).

As will be discussed, despite our high resolution, slight Gibbs ringing is intermittently observed at the smallest scales in the simulation, indicating that viscous dissipation is not entirely resolved (Lecoanet et al., 2016). To quantify the impact of this ringing, in addition to probing the sensitivity of our results to the domain size, we run a second simulation in a cubic domain with side length 4 and $N = 384$ modes, such that $\Delta x = \Delta y \approx 2\eta$. As shown in the appendix, no Gibbs ringing is apparent in this smaller simulation, and the time averages of the velocity field and plume radius in the larger domain and smaller domain agree to within 10% – an uncertainty factor that we will apply to our main results.

We employ the 2nd-order BDF scheme of Wang and Ruuth (2008) to solve the non-linear terms explicitly and the linear terms implicitly with a variable time step set by the Courant-Friedrichs-Lewy condition with a 0.25 safety factor. To achieve a steady state, we employ damping far from the source. Let $C(\hat{x}, \hat{y}, \hat{z})$ be a spatially-varying damping coefficient. Rayleigh drag ($-C\mathbf{u}$) and Newtonian cooling ($-Cb$) are then added to the momentum equation (Eqn. 4.12) and buoyancy equation (Eqn. 4.13), respectively. Damping is performed outside of a half ellipse centered at the surface with a vertical semi-major axis of 8 and a horizontal semi-minor axis of 4. A tanh function smoothly damps outside this ellipse:

$$C(\hat{x}, \hat{y}, \hat{z}) = \frac{1 + \tanh \left(\frac{f(\hat{x}, \hat{y}, \hat{z}) - H}{\Delta} \right)}{2\hat{\tau}_{\text{damp}}}, \quad (4.18)$$

$$f(\hat{x}, \hat{y}, \hat{z}) = \sqrt{2(\hat{x}^2 + \hat{y}^2) + \hat{z}^2}, \quad (4.19)$$

and we set the damping height as $H = 8$, transition length $\Delta = 1$, and time scale as $\hat{\tau}_{\text{damp}} = 1.5$. Simulations are initialized from rest with small-scale noise added to the starting buoyancy field (of magnitude $\hat{b} \sim 10^{-4}$) to break symmetry. The simulation is run for 40 time units. A statistical steady state is reached at $\hat{t} \approx 10$, and all analysis is performed on the last 30 time units of the simulation. Figure 4.1 plots instantaneous and time-averaged fields for this simulation.

In contrast to Plourde et al. (2008), no continuous source of artificial noise is needed in the simulations to coax the flow into turbulence. We note that Plourde et al. (2008) reports a diameter-based Reynolds number of 7700, which, upon converting to the radius-based definition (Eqn. 4.15) used here, yields $\text{Re} \approx 3055$. Prior experiments with $\text{Re} \sim 1000$ appear turbulent (as reviewed in Table 4.1) so the observation in Plourde et al. (2008) of a laminar plume at $\text{Re} \approx 3055$ is surprising. Plourde et al. (2008) view this laminarity as unrealistic, and the following analysis suggests that this laminarity may be due to under-resolving the flow.

The Kolmogorov length for $\text{Re} \approx 3055$ is $\eta/R = \text{Re}^{-3/4} \approx 1/400$. Plourde et al. (2008) simulated a $10R \times 10R \times 16R$ domain on a Cartesian grid with $720 \times 720 \times 1200$ points and uniform spacing in each dimension, yielding $\Delta x = \Delta y = \Delta z \approx 5\eta$. While this is comparable to the resolution used here, Plourde et al. (2008) employ a second-order central difference scheme rather than a spectral method, and finite differences generate significant numerical error. In particular, second-order central difference schemes with $\Delta x = 5\eta$ differentiate a wave with wavelength λ at 95% accuracy only if $\lambda \geq 57\eta$ (Moin and Mahesh, 1998). This analysis suggests that significant numerical error is present at inertial scales in Plourde et al. (2008) — consistent with their “unrealistically laminar” observation. Dedalus simulations run with fewer modes than the DNS requirement also appear more laminar in the sense that sharp gradients yield Gibbs waves rather than small-scale turbulent mixing. The result is a narrower plume with less turbulent entrainment (not shown here).

4.4 Results

We analyze here the results of the two-dimensional (2D) and three-dimensional (3D) DNS of a steady-state, turbulent plume sustained by a circular source of buoyancy at a no-slip surface in an unstratified, unsheared Boussinesq fluid.

2D simulations

The set of two-dimensional simulations (see Table 4.2) is designed to probe the sensitivity of z_v to a range of Re , varying from laminar to highly turbulent. These simulations are governed by the two-dimensional version of equations 4.12, 4.13, and 4.14 with a surface buoyancy source applied inside of $\hat{x} = 1$ (or inside $x = R$ with dimensional variables). First,

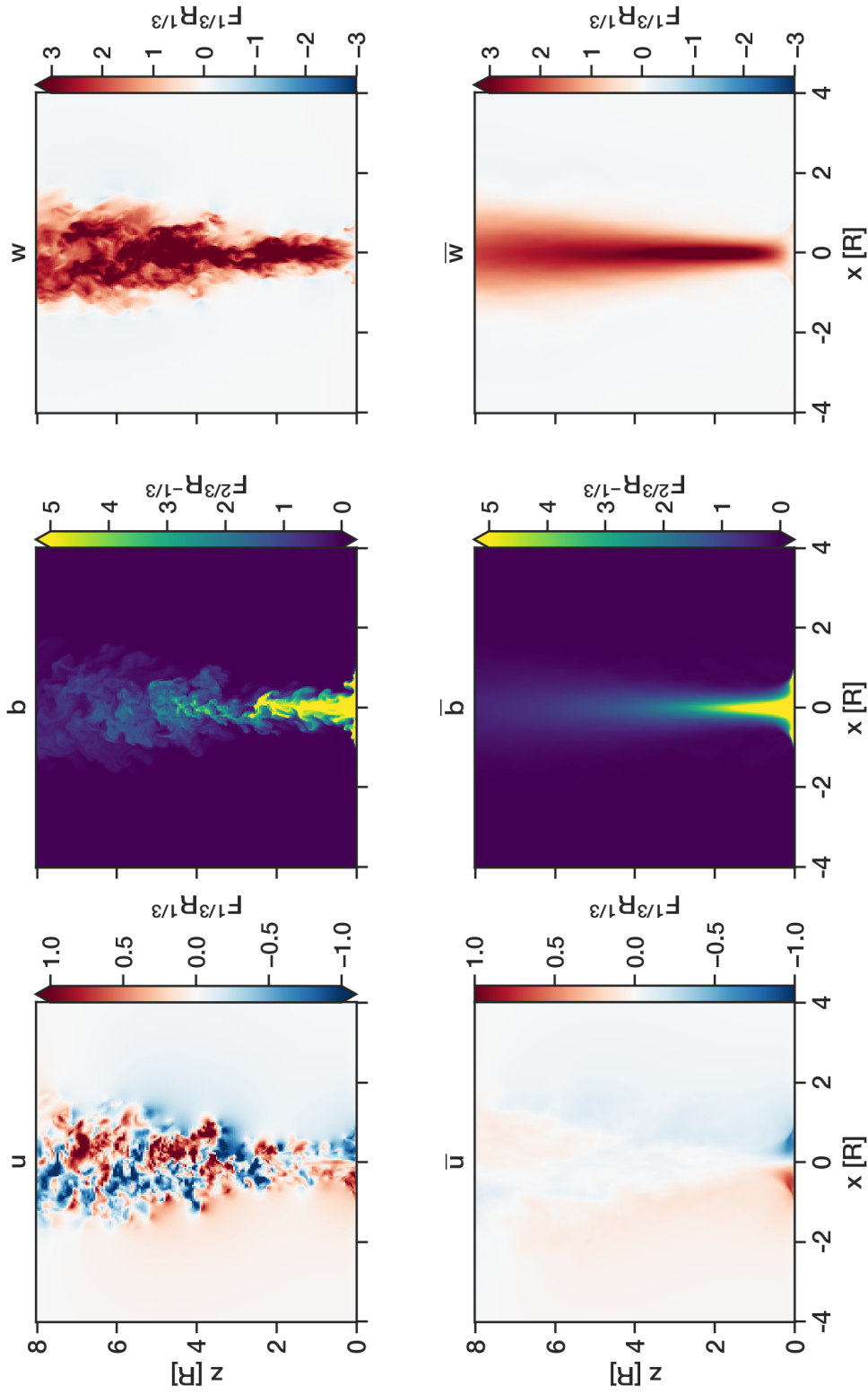


Figure 4.1: Instantaneous (top row) and time-averaged (bottom row) cross-sections of a three-dimensional turbulent plume with a Reynolds number of $Re = 10^3$ and Prandtl number of $Pr = .7$. Variables are non-dimensionalized in terms of the surface buoyancy source's radius R and magnitude F (with units of velocity $\times$ buoyancy).

we note that the two-dimensional analogues to the MTT equations (Eqn. 4.4, 4.5, 4.6) are

$$\frac{d}{dz}(rw) = \alpha w, \quad (4.20)$$

$$\frac{d}{dz}(rw^2) = rb, \quad (4.21)$$

$$rwb = Q = \text{constant}. \quad (4.22)$$

The solution with a point source of buoyancy at $r = 0$ and $z = z_v$ is $r = \alpha(z - z_v)$, $b \propto (z - z_v)^{-1}$, and w constant. To diagnose the simulated $r(z)$, we must confront the reality that plumes are not horizontally uniform like the bulk-plume (or top-hat) solution of MTT. Therefore, a functional definition of the plume's radius is required. Following convention (Morton et al., 1956; Kaye and Hunt, 2009; Bouzinaoui et al., 2007), we model the plume's radial distribution with a Gaussian. (In the next section, we verify this assumption and plot the Gaussian fit to the 3D data.) At each height, we fit a Gaussian to the time-averaged vertical velocity using least squares and identify the plume's effective radius as $r(z) = \sqrt{2\sigma(z)}$, where σ is the Gaussian's standard deviation. We can then approximate the time-averaged vertical velocity as

$$w(x, z) = w_c(z)e^{-x^2/r(z)^2}, \quad (4.23)$$

where $w_c(z)$ is the time-averaged center-line vertical velocity. The inferred $r(z)$ and center-line fields are plotted in Figure 4.2, and demonstrate a convergence towards a high-Re limit.

The values of α and z_v given in Table 4.2 are computed by a linear fit to the $r(z)$ curves far from surface. Given $\alpha = 0.24$, we also estimate the implied virtual origin of the ad hoc spun-up-at-surface assumption by solving for the z_v that enforces $r(0) = R$; that would yield $z_v = -4.12R$. Comparing this prediction to the results in Table 4.2, we reach two conclusions: 1. the high-Re plume spins up with a virtual origin that is an order of magnitude closer to the surface than in the spun-up-at-surface prediction, and 2. even for Re not yet converged to the high-Re limit, the spun-up-at-surface condition grossly overestimates the proper z_v . We emphasize this last point because the computational constraints of three-dimensional simulations prohibits us from probing a wide range of Re and formally demonstrating Re convergence in three dimensions. The two-dimensional analysis thus lends support to the notion that a $\text{Re} = 10^3$ simulation can be used to assess the accuracy of the spun-up-at-surface assumption in three dimensions.

3D simulations

In three dimensions, 10^3 was the highest achievable Reynolds number given the available computational resources, so a simulation with that Re is the one presented here. We identify $r(z)$ by again following the convention of fitting a Gaussian distribution to the plume's velocity field,

$$w(s, z) = w_c(z)e^{-s^2/r(z)^2}, \quad (4.24)$$

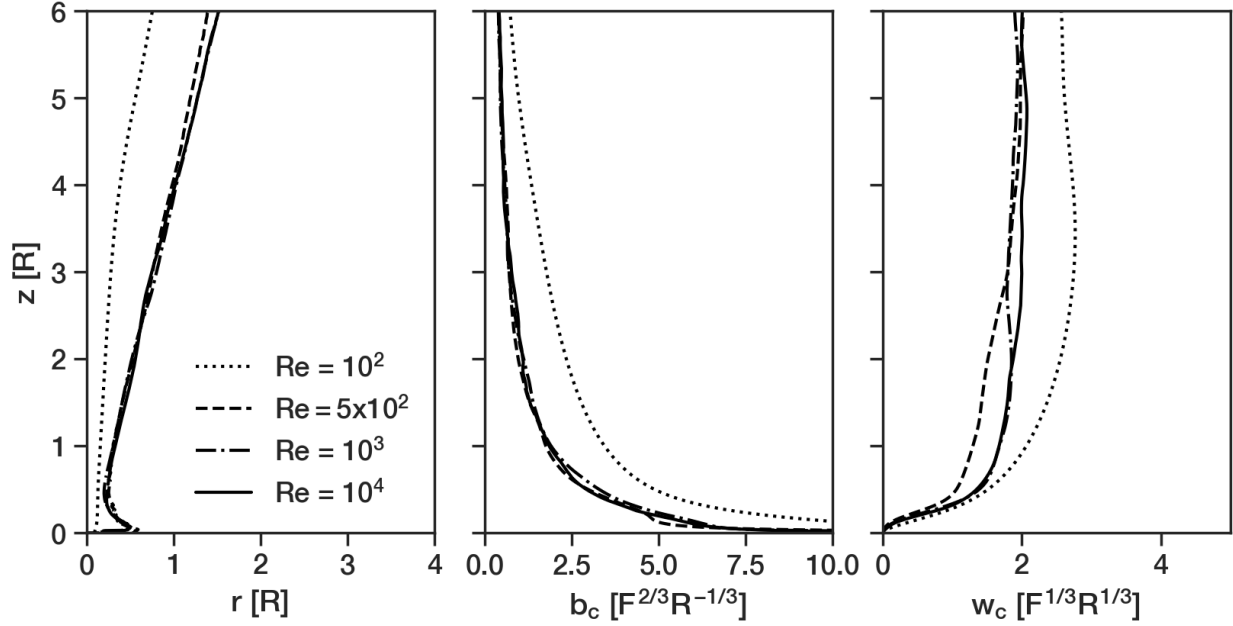


Figure 4.2: Time-averaged profiles of two-dimensional plumes with varying Re . The profiles of (left) radius, (middle) axial buoyancy, and (right) axial vertical velocity show a convergence toward an asymptotic turbulent limit.

Re	N modes	α	z_v [R]
10^2	256	N/A	N/A
5×10^2	384	.20	-0.94
10^3	512	.24	-0.30
10^4	1024	.24	-0.24

Table 4.2: Set of two-dimensional simulations performed. No z_v is fit to the $Re = 10^2$ case: this laminar plume grows diffusively and does not exhibit an MTT cone as shown in Figure 4.2.

where $s = \sqrt{x^2 + y^2}$ and $w_c(z)$ is the center-line vertical velocity. Figure 4.3 evinces the goodness of fit. In the left panel of Figure 4.4, we plot $r(z)$, which, regressed against z , yields an estimate of α and z_v via Eqn. 4.4. The regression is computed over $z = [3, 6]$, over which $r(z)$ appears approximately linear, providing $\alpha = .091 \pm .003$, in close agreement with prior studies (Morton et al., 1956; Carazzo et al., 2006), and $z_v = -(1.14 \pm .02)R$. The uncertainties given here are those stemming from the uncertainty of the least-squares fit.

Since we are not able to run the three-dimensional simulation over a wide range of Reynolds numbers, it is not possible to give a rigorous estimate of the deviation of $z_v = -1.14R$ from the true value of z_v in the high-Reynolds limit. By running a smaller domain and higher resolution simulation, we show in the appendix that the time-averaged velocity and radius have a roughly 10% sensitivity to the domain size and number of modes. In light of this sensitivity, we will quote the result to the closest tenth of R , i.e., $z_v = -1.1R$.

For context, we may get some sense for the Re dependence by looking at the 2D simulations, in which $|z_v|$ decreased by 20% from a Reynolds number of 10^3 to 10^4 . If 3D scales identically to 2D, we might expect the high-Re z_v to be slightly closer to the surface than the simulation presented here (run at a Reynolds number of 10^3).

For comparison, the practice of assuming an MTT plume that is spun-up at the surface yields $z_v = -5R/(6\alpha) = -9.2R$. The 3D simulation reveals that this estimate is off by nearly an order of magnitude. The consequences of this disagreement will be explored in the subsequent section. Encouragingly, our finding of $z_v = -1.1R$ lies within the range $z_v = [-.86, -2.2]$ identified in past experiments (Table 4.1). Note that we caution direct comparison, however, because these prior plumes are distinct from the case studied here, as they possessed either a laminar boundary layer, surface volume sources, or a free-slip boundary condition. Comparing to prior theoretical estimates, our result lies closer to the $z_v = 0$ prediction by Hunt and Kaye (2001) (obtained assuming a constant α all the way down to the surface) than to the $z_v = -3.34R$ prediction by Ciriello and Hunt (2020) (obtained using an α that is linear in the Richardson number).

Beyond z_v , we seek a theoretical prediction of the plume's vertical velocity and buoyancy. Formulating conservation equations for the Gaussian plume requires a slight generalization to account for the narrower buoyancy distribution, clearly visible in Figure 4.1 and 4.3. Readers are directed to section 1.2 of Hunt and Kaye (2001) for a review of past experimental work on this topic. While the radial distribution of velocity and buoyancy are both shaped by entrainment, we might expect vertical velocity to have a slightly different distribution due to the effects of pressure forces.

To account for buoyancy's narrower profile, we follow the standard practice reviewed in Turner (1986) of introducing a parameter λ such that

$$b(s, z) = b_c(z)e^{-s^2/\lambda^2 r(z)^2}. \quad (4.25)$$

By least-squares estimation, we find $\lambda = .89 \pm .07$ (compare to $\lambda = .83$ as reported in Bouzinaoui et al., 2007). Now that we have velocity and buoyancy distributions, we can integrate

over them to find the MTT conservation equations for a Gaussian plume (Bouzinaoui et al., 2007):

$$\frac{d(w_c r^2)}{dz} = 2\alpha r w_c, \quad (4.26)$$

$$\frac{d(\frac{1}{2}w_c^2 r^2)}{dz} = \lambda^2 b_c r^2, \quad (4.27)$$

$$\frac{\lambda^2}{\lambda^2 + 1} w_c b_c r^2 = Q, \quad (4.28)$$

which admit the solution,

$$r(z) = \frac{6\alpha}{5}(z - z_v), \quad (4.29)$$

$$w_c(z) = w_0 Q^{1/3} (z - z_v)^{-1/3}, \quad (4.30)$$

$$b_c(z) = b_0 Q^{2/3} (z - z_v)^{-5/3}, \quad (4.31)$$

where we have defined,

$$w_0 = \left(\frac{6\alpha}{5}\right)^{-2/3} \left(\frac{3(\lambda^2 + 1)}{2}\right)^{1/3}, \quad (4.32)$$

$$b_0 = \left(\frac{2}{3\lambda^2}\right) \left(\frac{6\alpha}{5}\right)^{-4/3} \left(\frac{3(\lambda^2 + 1)}{2}\right)^{2/3}. \quad (4.33)$$

Given the best-fit parameters collected in Table 4.3, we proceed to computing $w_c(z)$ and $b_c(z)$ as shown in Figure 4.4. Beneath $z \approx 2R$, the plume fields substantially diverge from the MTT scalings: the vertical velocity goes to zero to match the no-flux surface boundary condition, and the buoyancy field transitions to a conductive layer near the surface. This behavior is manifestly different from the near-surface MTT scalings plotted in dashed lines in Figure 4.4. Above this surface layer, however, the MTT theory is accurate to within 10%, and can be readily employed to estimate the plume's properties. We note that results presented here are non-dimensional and are thus applicable to arbitrary radius R and buoyancy flux F . Numerical values of r , b_c , and w_c at heights below $2R$ are given in Table 4.4. Taken in combination with the MTT solution constrained by the parameters in Table 4.3, this amounts to a general and comprehensive description of all sufficiently turbulent Boussinesq plumes rising above uniform circular sources of buoyancy.

4.5 Hiroshima fire plume

We are now in a position to calculate the errors generated by the ad hoc closures of 1. placing the virtual origin at the surface (i.e., $z_v = 0$) and 2. assuming the MTT plume is spun-up at

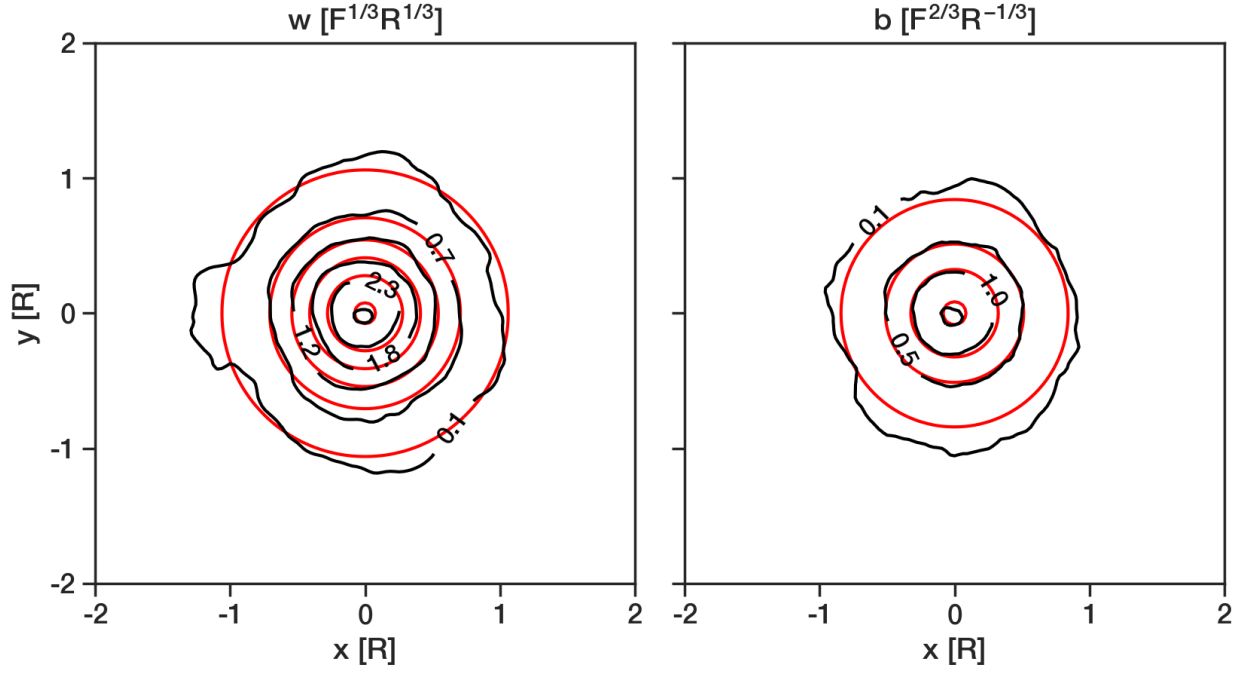


Figure 4.3: Horizontal distribution of time-averaged vertical velocity (left) and buoyancy (right) at $\hat{z} = 5$ in the $Re = 10^3$ simulation (black) and the corresponding least-squares fit (red) to the Gaussian distributions in Eqn. 4.25.

	b_0	w_0	α	λ	$z_v [R]$
Central Est.	31.29	6.08	.091	.89	-1.14
Uncert.	3.64	.15	.0003	.073	.02

Table 4.3: Best-fit parameters to Gaussian model of three-dimensional plume.

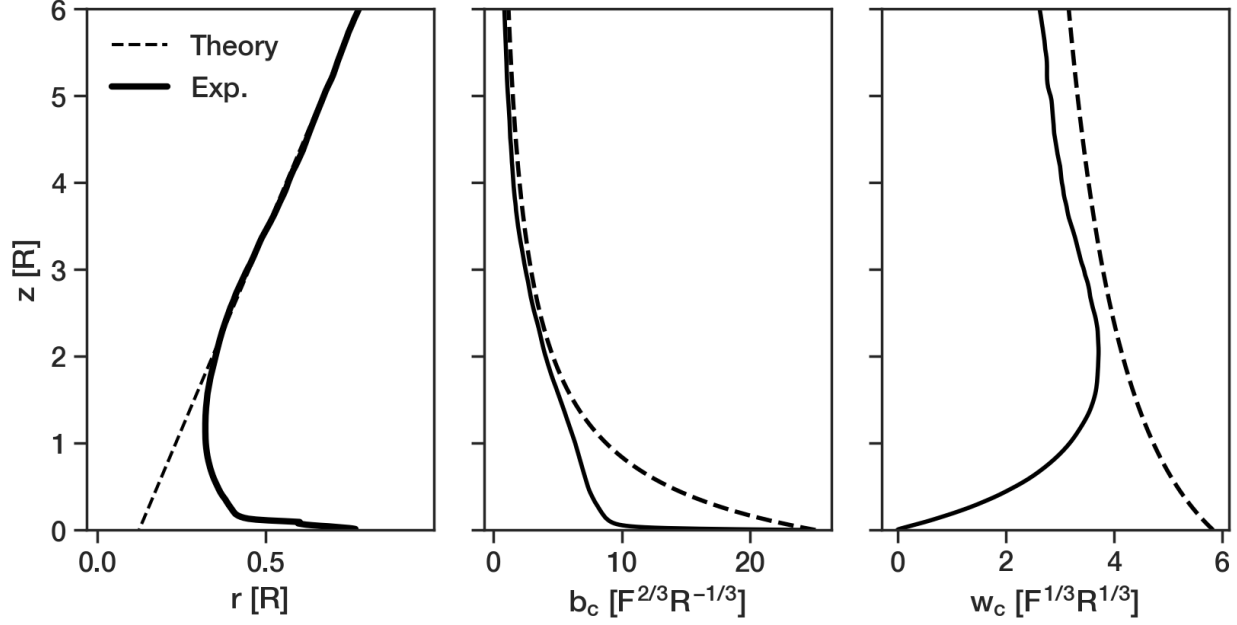


Figure 4.4: Time-averaged profiles from (solid) the simulations and (dashed) equations (4.29-5.6) of (left) the plume's radius, (middle) axial vertical velocity, and (right) axial buoyancy.

$z [R]$	$r [R]$	$b_c [F^{2/3}R^{-1/3}]$	$w_c [F^{1/3}R^{1/3}]$
0	.68	24.38	0.0
.25	.4	8.22	1.29
.5	.36	7.36	2.19
.75	.33	6.88	2.76
1.0	.32	6.38	3.18
1.25	.32	5.79	3.45
1.5	.33	5.21	3.63
1.75	.34	4.63	3.69
2.0	.35	4.09	3.71

Table 4.4: The high-Re DNS solution for the spin-up of the MTT plume plotted in Figure 4.4.

the surface with a width equal to that of the buoyancy source (i.e., $z_v = -5R/(6\alpha) = -9.2$). Noting that the numerical solution asymptotes to an MTT solution at about $z = 2R$, we can compare the predictions that those closures would make for the center-line buoyancy at $z = 2R$ relative to the actual buoyancy. Noting that $b_c(z, z_v) \propto (z - z_v)^{-5/3}$, we find

$$\frac{b_c(z = 2R, z_v = 0)}{b_c(z = 2R, z_v = -1.1R)} = \left(\frac{2}{2 + 1.1} \right)^{-5/3} = 2.0 \quad (4.34)$$

$$\frac{b_c(z = 2R, z_v = -9.2R)}{b_c(z = 2R, z_v = -1.1R)} = \left(\frac{2 + 9.2}{2 + 1.1} \right)^{-5/3} = 0.12. \quad (4.35)$$

In other words, the two previously used closures overestimate and underestimate the plume buoyancy by factors of 2.0 and 0.12, respectively.

In the atmosphere, we are often most interested in a plume's level of neutral buoyancy since that determines the height of the layer of concentrated smoke, ash, or other pollutants. For a buoyant plume in an unstratified fluid, however, there is no level of neutral buoyancy: the plume remains more buoyant than its surroundings at all heights. Thus, the MTT theory – and the closure provided for it in Table 4.3 – is not directly applicable to plumes outside of a well-mixed layer. On the other hand, we expect that plumes are largely unaffected by environmental stratifications $|\partial b_{\text{env}}/\partial z|$ that are small compared to the plume's own buoyancy gradient $|db_c/dz|$. Therefore, the high-Re solution given in Figure 4 and Table 4.4 should apply to plumes in a stratified atmosphere near the surface. For this reason, it is likely that the virtual origin of a plume in Earth's stratified atmosphere is nearly identical to the value of $-1.1R$ found here.

Away from the surface, the solution asymptotes to an MTT plume, which is highly constrained by dimensional analysis. By Equation 4.1, the MTT solution is simply a plume of volume rate $V = \pi r^2 w$. Therefore, the fractional entrainment rate ε for the MTT solution is $\varepsilon \equiv d \log V / dz = (2/r) dr / dz = 2\alpha/r$, where $2\alpha \approx 0.2$. As mentioned earlier, this $1/r$ scaling stems from the reasonable assumption that the eddies responsible for entrainment scale with the size of the plume. Indeed, such $1/r$ scaling has been observed in many other contexts as well, including clouds (Stirling and Stratton, 2012a) and isolated thermals (Lecoanet and Jeevanjee, 2019a). Below $z = 2R$, ε deviates from this simple $1/r$ scaling, but its value can be calculated at each height from the evolution of the center-line buoyancy (see below), allowing for the calculation of the full profile of $\varepsilon(z)$.

We can approximate a plume in a stratified atmosphere using the initial conditions and the $\varepsilon(z)$ from the simulation of the unstratified case. For the initial conditions, we take the buoyancy and vertical velocity of the DNS solution at the surface. For adiabatic ascent without entrainment, the dry static energy DSE (equal to $c_p T + gz$) of a lifted parcel is approximately conserved¹. Parameterizing entrainment as a relaxation of the plume's DSE

¹DSE - CAPE is the true conserved variable for a lifted adiabatic dry parcel (Romps, 2015). Consider ascent over a height H with characteristic buoyancy B . Comparing $\text{CAPE} \sim BH$ and the DSE anomaly

to the environmental value over the mixing length $1/\epsilon$ yields

$$\frac{d\text{DSE}}{dz} = \epsilon(\text{DSE}_e - \text{DSE}), \quad (4.37)$$

where the subscript e denotes environmental values. In addition, we solve an analogous equation for the water-vapor mass fraction q_v to ensure that the solution is terminated when the plume begins to condense:

$$\frac{dq_v}{dz} = \epsilon(q_{ve} - q_v). \quad (4.38)$$

While follow-up research is warranted to quantify the impact of stratification on plume solutions, the point we wish to make is in no way subtle, and so this approach will suffice.

In particular, we wish to assess the appropriateness of the spun-up-at-surface approximation in application to the Hiroshima firestorm of August 6, 1945. Well after the mushroom cloud from the nuclear detonation had dissipated, the citywide fire sourced a steady plume of radioactive soot. The fate of soot from such storms is of great practical importance considering the potential for deposition of radioactive material far from the blast site and the potential for soot to enter the stratosphere where it can perturb Earth's climate. To model the fate of that soot, however, it is critical to know the plume's level of neutral buoyancy.

For the Hiroshima firestorm, historical estimates give $R \approx 1$ km and a surface heat flux of 20 kW/m^2 (Aoyama et al., 2011), yielding a buoyancy flux of

$$F = \frac{g}{\rho c_p \theta_0} (20 \text{ kW m}^{-2}) \approx 0.7 \text{ m}^2 \text{ s}^{-3}, \quad (4.39)$$

where we have let $\theta_0 \approx 300 \text{ K}$, $\rho \approx 1 \text{ kg m}^{-3}$, and $c_p \approx 1000 \text{ J kg}^{-1} \text{ K}^{-1}$. To make contact with the DNS results, we compute the dimensionful scale $T = F^{-1/3} R^{2/3} \approx 100 \text{ s}$. In roughly $10T \approx 20$ minutes, the plume ascends from the surface to near the domain top at $10R = 10 \text{ km}$ and the simulation achieves steady-state. A steady-state treatment of the Hiroshima plume appears reasonable given that observations show that the firestorm burned for several hours (Aoyama et al., 2011). The velocity scale of the bulk plume is $U = F^{1/3} R^{1/3} \approx 9 \text{ ms}^{-1}$ with max vertical velocities of $w_c \approx 4U \approx 40 \text{ ms}^{-1}$ occurring near 2 km as shown in Figure 4.4. Axial buoyancies in the mature plume are in excess of the buoyancy scale $B = F^{2/3} R^{-1/3} \approx .07 \text{ ms}^{-2}$, but share the same order of magnitude above the first hundred meters.

Following the analysis in Tajima (1983), we employ the sounding of Tokyo reported in Kerr et al. (1983) as a proxy for Hiroshima's atmosphere on the morning of August 6, 1945. The sounding, plotted as circles in Figure 4.5, is consistent with qualitative observations

$\text{DSE}' \sim c_p B T_e / g$, we find

$$\frac{\text{CAPE}}{\text{DSE}'} \sim \frac{gH}{c_p T_e} \sim 10^{-1}, \quad (4.36)$$

where we have plugged in with $H = 10 \text{ km}$. We therefore take DSE' to be approximately conserved.

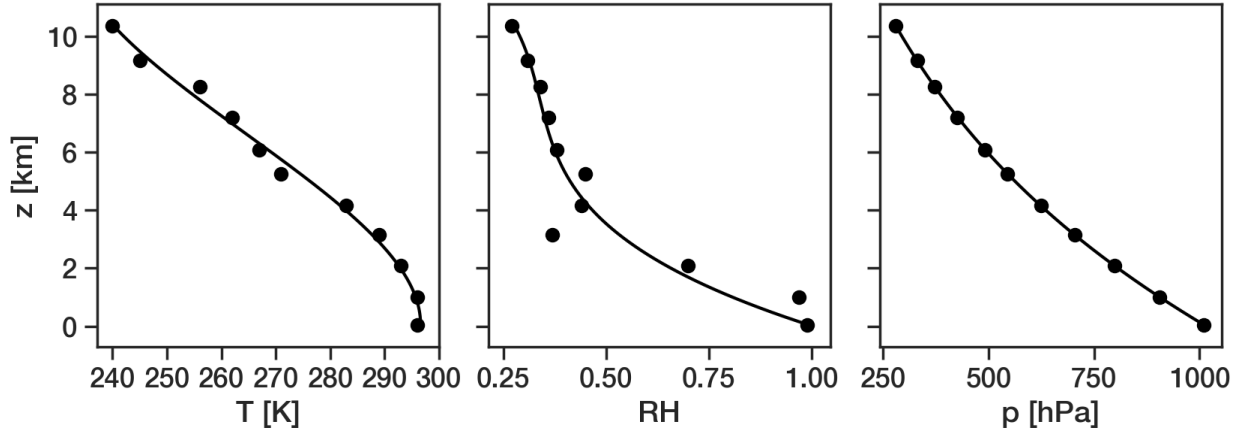


Figure 4.5: Hiroshima atmospheric state: temperature, relative humidity, and pressure as estimated from the atmospheric sounding of Tokyo on August 6 at 6:00am local (21:00 UCT, August 5) measured by US strategic bombing command (Kerr et al., 1983). Circles show the sounding and the curves show third-degree polynomial fits.

describing “Japan [as] widely covered homogeneously by a hot and humid maritime air mass” (Tajima, 1983). We fit third-degree polynomials to these soundings, shown as solid curves in Figure 4.5.

To predict the fire plume’s level of neutral buoyancy, we can model the plume as an MTT plume spun-up at the surface (which we will denote by a subscript “MTT”) or we can model the plume using the solution found in the previous section (which we will denote by a subscript “DNS”). As we will see momentarily, the initial conditions and the entrainment profiles differ dramatically between these two cases.

Using equations (4.29), (5.6), and (4.33), we can calculate the initial centerline buoyancy for the spun-up-at-surface MTT solution as

$$b_{c,\text{MTT}}(0) = \left(\frac{2}{3\lambda^2} \right) \left(\frac{6\alpha}{5} \right)^{1/3} \left(\frac{3(\lambda^2 + 1)}{2} \right)^{2/3} Q^{2/3} R^{-5/3} = 0.06 \text{ m s}^{-2}, \quad (4.40)$$

where the values of α and λ are taken from Table (4.3), and $Q = FR^2 = 7 \times 10^5 \text{ m}^4 \text{ s}^{-3}$. Note that a buoyancy of 0.06 m s^{-2} corresponds to a temperature anomaly of 2 K. The fractional entrainment rate ε can be found from the dilution of the centerline buoyancy,

$$\varepsilon_{\text{MTT}}(z) = -\frac{d \log b_{c,\text{MTT}}(z)}{dz} = \frac{5}{5R/2\alpha + 3z}. \quad (4.41)$$

This entrainment rate is plotted in red in the left panel of Figure 4.6. At the surface, this fractional entrainment rate equals 0.2 km^{-1} .

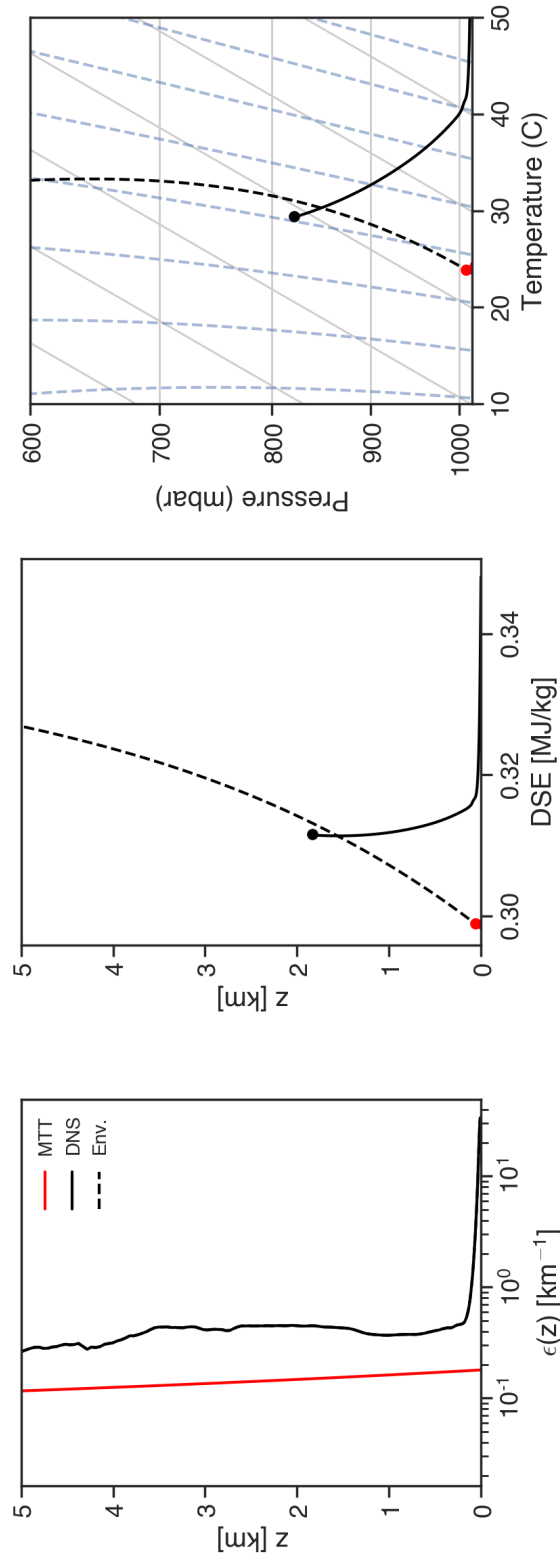


Figure 4.6: (left) Fractional entrainment rate ϵ for the (red) spun-up-at-surface MTT (red) and (black) DNS solutions. (middle) Profiles of buoyancy up to the lifting condensation level (marked by a circle) using the initial conditions and entrainment rates appropriate to the spun-up-at-surface MTT and DNS cases. (right) Skew-T log-p diagram of the (dashed) environment and (solid) parcel trajectories.

For the DNS plume, the time-averaged surface buoyancy at the center is

$$b_{c,\text{DNS}}(0) = 24.38F^{2/3}R^{-1/3} = 2 \text{ m s}^{-2}, \quad (4.42)$$

which corresponds to a temperature anomaly of 60 K. The fractional entrainment rate can be found from the dilution of the centerline buoyancy,

$$\varepsilon_{\text{DNS}}(z) = -\frac{d \log b_{c,\text{DNS}}(z)}{dz}. \quad (4.43)$$

This is plotted as the black curve in the left panel of Figure 4.6. Very close to the surface, where conduction is dominant, this is not technically the entrainment rate. In fact, at the surface, this definition gives $\varepsilon_{\text{DNS}}(0) = F/\kappa/b_{c,\text{DNS}}(0)$, which represents pure conduction. Above the non-dimensional height where $\hat{b}\hat{w} = \text{Re}^{-1}\text{Pr}^{-1}d\hat{b}/d\hat{z}$, advection of buoyancy dominates over conduction. For the DNS solution, this occurs at a height of .015; well above this height, it is correct to interpret the ε_{DNS} defined in equation (4.43) as the fractional entrainment rate.

We can integrate the buoyancy (initialized to its surface MTT or DNS value) and water-vapor mass fraction (initialized to the environmental value) using equations (4.37) and (4.38), the MTT and DNS entrainment rates in equations (4.41) and (4.43), respectively, and the polynomial fits to the soundings for the environmental values of DSE and q_v . The resulting profiles of buoyancy for the MTT and DNS plumes are plotted in the middle panel of Figure 4.6 up to their lifting condensation levels (LCLs). As a result of the stable conditions and the MTT parcel's unremarkable buoyancy, the lifting calculation pegs the spun-up-at-surface MTT LCL to within 100 m of the surface. Given the environmental convective inhibition (CIN), we conclude that the spun-up-at-surface plume will not reach the upper atmosphere. In contrast, the DNS parcel achieves an LCL of roughly 2 km. Will the DNS parcel reach the upper atmosphere? CIN above the LCL complicates the answer, which depends on the parcel's momentum and moist ascent. A naive estimate of $w_{\text{LCL}} = (2 \int b dz)^{1/2} \approx 30 \text{ m/s}$ suggests that the parcel may overcome the overlying CIN $\approx 100 \text{ J/kg}$ even without additional buoyancy from latent heating. Regardless, the DNS parcel's trajectory is markedly different than the spun-up-at-surface MTT parcel, which would predict that all of Hiroshima's hot gases and radioactive soot would remain trapped near the surface. For comparison, computer vision analysis of Hiroshima firestorm photographs suggests that the plume extended up to a height of 16 km (Baba et al., 2011). Lifting the parcel beyond the LCL and comparing the moist level of neutral buoyancy to this observation requires an account of moist entrainment and microphysics, which is beyond the scope of this paper.

4.6 Conclusion

Turbulent plumes (sustained by a circular source of buoyancy at a no-slip lower boundary in an unsheared, unstratified fluid) exhibit a distinct spin-up stage before they transition to

the scalings predicted by Morton et al. (1956, MTT). Two facts – the rapid convergence of solutions to the high-Re limit and the approximate invariance of plume solutions with respect to the Prandtl number – lead to a remarkable simplification of the solution set of turbulent plumes. Indeed, postulating invariance with respect to both Re and Pr in the high-Re limit, there are no remaining free parameters. This suggests that the DNS solution presented here (see Figure 4.4) is the one and only solution for all such turbulent plumes. Although we have presented evidence from two-dimensional solutions that this three-dimensional simulation is plausibly converged to the high-Re limit at a Reynolds number of only 10^3 , simulations or experiments at higher Reynolds numbers should be performed to confirm this.

Well above the surface, all plumes (in an unstratified, unsheared fluid) asymptote to the MTT solutions. Since the ratio of the entrainment velocity to the updraft speed (α) rapidly converges to about 0.1 in the high-Reynolds limit, the dimensional MTT solutions are uniquely defined by only two parameters: the buoyancy source rate Q and the height of the virtual origin z_v . Since Q is typically a given, this leaves only z_v to be determined.

The three-dimensional DNS solution obtained here establishes that the plume’s virtual origin is substantially different from the values previously used. Previous work has often assumed that the plume is spun up to an MTT solution at the surface with a width equal to the width of the buoyancy source. That spun-up-at-surface assumption places the virtual origin of the MTT plume below the surface a distance about ten times the radius of the buoyancy source. In contrast, the DNS solution finds a virtual origin that lies below the surface by a distance equal to 1.1 times the source radius (see Table 4.3). Consequently, the spun-up-at-surface approximation implies a buoyancy at a height of $2R$ (which is where the DNS solution asymptotes to the MTT solution) that is 10 times too small (see equation 4.35).

We have applied the DNS solution to the case of the Hiroshima firestorm to predict the level of neutral buoyancy of the generated plume. The spun-up-at-surface approximation would predict that the radioactive soot would be trapped within a few hundred of meters of the surface. In contrast, using the entrainment rate obtained from the DNS solution, we predict that the plume reached the free troposphere, and possibly even the upper atmosphere (Figure 4.6) in agreement with observational evidence (Baba et al., 2011).

The last result is of particular importance because soot in the upper atmosphere — suspended above the weather layer — may remain aloft for months to years. Spread across the globe by the stratospheric circulation, this upper atmospheric soot can produce substantial surface cooling. In the nuclear firestorm context, this cooling may induce a so-called nuclear winter (see, e.g., Toon et al., 2007). Thus, an accurate estimate of the plume’s virtual origin is essential to predicting the plume’s upper-atmospheric fate.

Appendix

Here we describe a smaller-domain simulation run to assess the impact of the slight Gibbs ringing and domain-size effects in the larger three-dimensional simulation described in Section 4.3. We employ a cubic domain of side length $4R$ with $N = 384$ modes (compare to the $12R$ length and $N = 512$ modes of the larger simulation). At the same Re of 10^3 , the smaller domain has $\Delta x = 2\eta$ while the larger simulation has $\Delta x = 4\eta$. All other aspects of the numerical setup are the same except for the damping layer, which is described by equation 4.19 but with a smaller height $H = 2.5$, transition length $\Delta = .25$, and $\tau_{\text{damp}} = 1/3$. The simulation is run for $20T$ and time averages are performed over the last $10T$ to compute the mean fields.

Figure 4.7 compares this smaller domain simulation to the larger simulation. The same larger simulation data plotted in Figure 4.1 is shown again in Figure 4.7, but with a different color scheme that emphasizes the slight ringing — an order of magnitude smaller than the mean fields. Thanks to the exponentially fast convergence of spectral DNS (Lecoanet et al., 2016), increasing the resolution eliminates any visible signs of Gibbs ringing, indicating that the small domain simulation is well converged. The bulk geometry of the plumes and the magnitudes of the vertical velocity are in broad agreement between the two simulations. The larger domain simulation, however, exhibits a roughly 10% higher peak velocity. Both the domain size and resolution are different, so the exact source of this discrepancy is unclear. We leave that question to future work, and, for the purposes of this paper, we take 10% to be a rough estimate of the uncertainty in the larger simulation results.

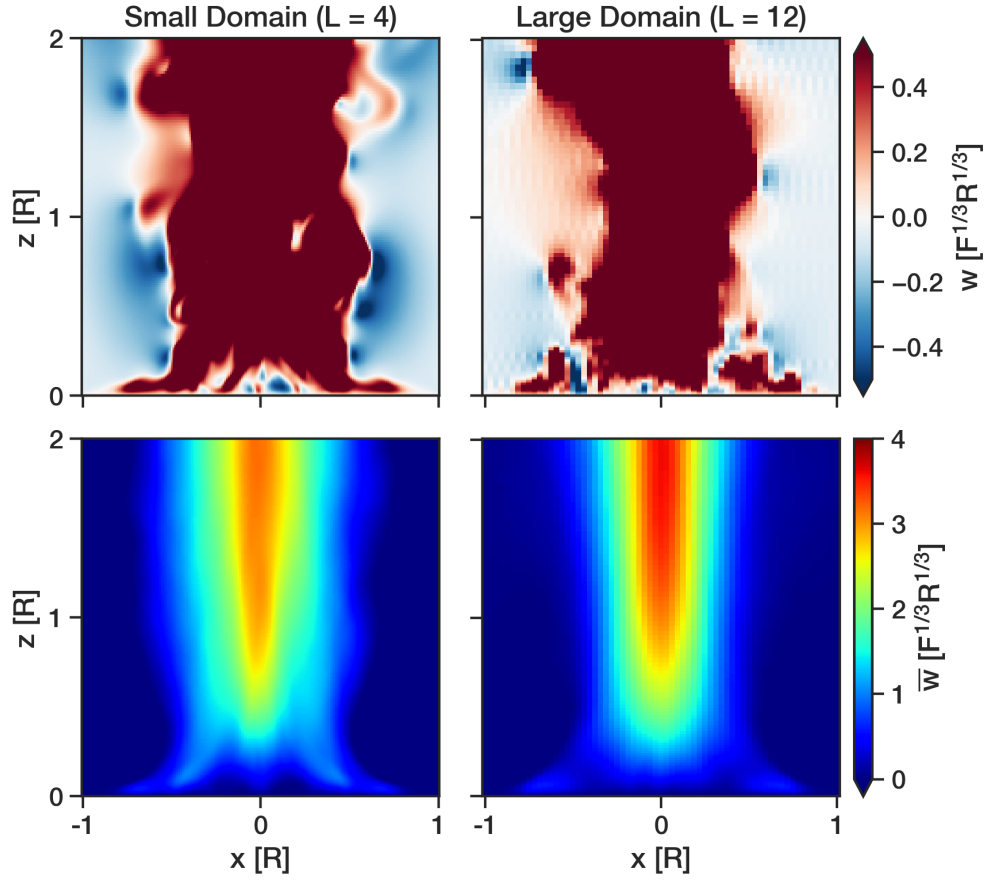


Figure 4.7: Top row: near-source instantaneous vertical velocity in the small domain (left) and large domain (right). Large domain data is a subregion of Figure 4.1 plotted with a color scheme that highlights the wave-like ringing rather than the mean fields. Bottom row: comparison of the time-averaged vertical velocities. See text for discussion.

Chapter 5

Latent heating is required for firestorm plumes to reach the stratosphere

City-wide firestorms produce extreme convective plumes that loft soot into the atmosphere. If emplaced in the stratosphere, soot has long-lasting impacts on global climate. Given the extreme sensible heating from the fire, the importance of additional heating from condensation is unclear and a subject of debate. Analytic plume calculations presented here establish that an idealized dry plume requires a temperature anomaly of at least 60 K at the top of the boundary-layer to remain buoyant up to the cold-point tropopause. Direct numerical and large-eddy simulations indicate that dry firestorm plumes possess temperature anomalies that are less than the requirements for stratospheric ascent by a factor of two or more. In contrast, moist firestorm plumes are shown to reach the stratosphere by tapping into the abundant latent heat present in a moist environment. Latent heating is found to be essential to plume rise, raising doubts about the applicability of past work that neglected moisture.

5.1 Introduction

The bombings of Hiroshima, Hamburg, and Dresden ignited city-wide fires intense enough to generate their own mesoscale atmospheric circulations (Manins, 1985). These circulations, known as firestorms, act as atmospheric bellows, fanning the fire to even greater intensity by drawing in surrounding air. If triggered, a firestorm is likely to consume all available fuel, and in doing so, firestorms transform cities into vast quantities of soot (Carrier et al., 1985). For context, the Hiroshima firestorm is estimated to have released ten times more energy than

The content of this chapter was previously published in Tarshish and Roms (2022).

the detonated atomic bomb (Aoyama et al., 2011; Toon et al., 2007). The fate of firestorm soot has profound consequences for global climate. If lofted by the fire to the stratosphere, soot will block sunlight from reaching the surface and stay lofted for years. Climate models find that megatons of stratospheric soot generate a “nuclear winter” by lowering the global temperature by several degrees Celsius over the following decade (Turco et al., 1983; Robock et al., 2007; Mills et al., 2014; Coupe et al., 2019; Jägermeyr et al., 2020).

Recent work has aimed to understand what scale of modern nuclear conflict would yield this magnitude of stratospheric soot by studying large eddy simulations of plume ascent (Reisner et al., 2018; Wagman et al., 2020). A notable difference between two recent studies is that Reisner et al. (2018) ran a dry simulation (i.e., without condensation and latent heating) of the firestorm plume, while Wagman et al. (2020) ran a moist simulation. In ordinary convection, latent heating is the dominant source of buoyancy that lifts deep plumes to the upper atmosphere, but whether latent heating is essential to firestorm plumes is under debate (Robock et al., 2019; Reisner et al., 2019). Robock et al. (2019) emphasized that large wildfire pyrocumulonimbi are moist, suggesting latent heating’s relevance for urban firestorms. On the other hand, Reisner et al. (2019) maintained that wildfire pyrocumulonimbi are exceptional cases, and that extrapolation to firestorms requires further research.

The exceptional formation of a pyrocumulonimbus is known to be dependent on the abundance of environmental moisture (Trentmann et al., 2006; Lareau and Clements, 2016). In observations, the availability of mid-tropospheric moisture is a determining factor of whether a wildfire will generate a deep convective plume (Peterson et al., 2017). Stratospheric soot injection from wildfires relies on the buoyancy supplied by condensing this environmental moisture (Trentmann et al., 2006; Peterson et al., 2021).

Differences in geometry and fuel load between wildfires and urban firestorms, however, make dry stratospheric ascent more plausible for a firestorm plume. Urban environments can possess an order of magnitude greater fuel load than dense forest, yielding fire intensities not seen in natural landscapes (Penner et al., 1986; Redfern et al., 2021). Simultaneous ignition of a compact area by a nuclear weapon also yields a fire geometry that contrasts with the perimeter or frontal structure typical of wildfires, and compact areal fires are more favorable to deep convection (Badlan et al., 2021). Therefore, whether moisture is required for stratospheric firestorm plumes remains unclear.

Early modeling efforts addressed this question to two-dimensional simulations and found conflicting results. Small and Heikes (1988) found that a dry firestorm plume lofted soot to the stratosphere. The dry simulations in Penner et al. (1986), on the other hand, run in a similar environment and with comparable fires, became neutrally buoyant before the tropopause. Prior to these numerical models, scaling theories and observation-based models did not emphasize the role of moisture, highlighting instead that the leading determinants of plume height were the magnitude of the fire’s heat fluxes and the degree of turbulent mixing with the environment (Manins, 1985; Carrier et al., 1985). In contrast, recent large-eddy simulations in Wagman et al. (2020) found that turning off latent heating lowered the height of soot accumulation from the tropopause at ~ 15 km down to 8 km, suggesting that moisture

	$\mathcal{H}$ [kW m ⁻²]	R [km]
Hiroshima Aoyama et al. (2011)	20	1
Hamburg Carrier et al. (1985)	70	2
Penner et al. (1986)	10, 90	5
Wagman et al. (2020)	10, 50, 100, 160	2
Small and Heikes (1988)	50	7
Reisner et al. (2018)	~40	~2

Table 5.1: Firestorm sources in the literature.

indeed plays a fundamental role in soot lofting.

Redfern et al. (2021) also investigated this question by simulating large fires in the same environmental conditions as Reisner et al. (2018), but with the addition of moisture. However, none of the simulations in Redfern et al. (2021) developed into a firestorm. Due to model limitations, the fire source was taken to be a heavy logging slash bed instead of an urban environment (Redfern et al., 2021). In addition, the dynamic fire model did not generate a quasi-steady large area source; instead, all of the fuel in the ignition area was consumed in the first few minutes, and the blaze quickly subsided into a much weaker perimeter fire (Redfern et al., 2021). Absent the mesoscale circulation of a firestorm, and given the fuel load, these results more closely resemble wildfires.

Our lack of a systematic understanding of the effects of latent heating on firestorm plumes presents a challenge to synthesizing these prior findings. It remains unclear under what circumstances, if any, moisture can be relegated to a second-order role. A complicating factor is that firestorm source conditions vary from study to study, as reviewed in Table 1. Fire radii R vary between 1 and 10 km and sensible heat fluxes are generally within the range $\mathcal{H} = 10 - 100$ kW m⁻². It is possible that the importance of latent heating is not uniform across this regime.

Here we seek to clarify the importance of latent heating for firestorm soot lofting by investigating the following questions:

- How hot must a plume be at the top of the boundary layer to rise to the stratosphere under dry ascent? (See section 5.2.)
- Do plumes generated by urban firestorms get this hot? Given heat fluxes in the range of $\mathcal{H} = 10-100$ kW m⁻², it may seem intuitive that the plume could be made increasingly hot by enlarging the fire radius R , but we will see that is not true. (See section 5.3.)

Finally, we probe the importance of latent heating by simulating firestorms in the Reisner et al. (2018) environmental atmosphere with and without moisture in section 5.4.

5.2 What plume temperature at the boundary-layer top enables dry ascent to the stratosphere?

We will quantify here the temperature anomaly of a plume entering the troposphere that is required for buoyant ascent to the tropopause without the aid of latent heating.

Lifting an adiabatic parcel

To facilitate a first estimate of plume rise, we idealize the plume as a parcel and compute the parcel's level of neutral buoyancy under dry, adiabatic ascent. Let T'_0 be the plume's temperature anomaly as it enters the free troposphere at a height of $z_0 \approx 1$ km. As a first pass, we approximate the atmosphere as having a constant lapse rate of $\Gamma = 6$ K km⁻¹. Dry adiabatic ascent sets the parcel's dT/dz to $g/c_p \approx 10$ K km⁻¹, where g is the gravitational constant and c_p is the specific heat capacity of air. For every 1 K of temperature anomaly at z_0 , the parcel's level of neutral buoyancy (LNB) rises by $(1\text{K})/(g/c_p - \Gamma) = 250$ m. In the tropical atmosphere, where the tropopause has a height near 15 km, we conclude that an initial temperature anomaly near 60 K is required for dry, adiabatic ascent to the stratosphere.

Lifting an entraining parcel

We expect that this adiabatic calculation underestimates the initial temperature required for upper atmospheric ascent because we neglected turbulent mixing, which dilutes the plume as it ascends. We approximate this mixing process by assuming there is a characteristic mixing length ℓ over which the parcel's dry static energy s relaxes to the environmental dry static energy s_e such that

$$\frac{ds}{dz} = \frac{(s_e - s)}{\ell}. \quad (5.1)$$

Integrating the dry static energy equation for a plume with initial temperature anomaly T'_0 in an environment with constant lapse rate Γ yields

$$T'(z) = e^{-(z-z_0)/\ell} T'_0 + (1 - e^{-(z-z_0)/\ell}) (\Gamma - g/c_p) \ell. \quad (5.2)$$

Let T_0^* be the temperature anomaly at z_0 required for the plume to reach a level of neutral buoyancy at the tropopause z_t . Setting $T'(z_t) = 0$ and solving for T_0^* yields

$$T_0^* = (e^{(z_t-z_0)/\ell} - 1) \left(\frac{g}{c_p} - \Gamma \right) \ell, \quad (5.3)$$

which reveals that T_0^* strongly depends on the value of ℓ when $z_t - z_0 \gg \ell$.

To make progress and enable a rough estimation, we adopt the standard Morton, Taylor, and Turner (1956) (hereafter, MTT) entrainment closure, which assumes that the mixing

length scales with the horizontal extent of the plume. In other words, the largest vortices, of size comparable to the plume itself, govern the mixing of ambient fluid. Given the plume's radius r , the MTT closure is $\ell = r/(2\alpha)$, where the non-dimensional entrainment constant α has been measured to be near 0.1 in a variety of convection experiments (Tarshish and Romps, 2021; Lecoanet and Jeevanjee, 2019b; Stirling and Stratton, 2012b; Carazzo et al., 2006). Taking $\alpha = 0.1$ (i.e., $\ell \approx 5r$) and approximating the plume's radius r with the source's radius R provides a simple closure of $\ell = 5R$ that enables analytical computation. For $R = 1\text{-}10$ km (the approximate horizontal extent of a city), plugging into Eqn. 5.3 yields a first estimation of the temperature required of the entraining plume as it enters the free troposphere if it is to reach the stratosphere. For a tropopause of $z_t = 15$ km, lapse rate $\Gamma = 6 \text{ K km}^{-1}$, and starting parcel height of $z_0 = 1$ km, Eqn. 5.3 yields the solid curve shown in Figure 5.1.

We observe that the importance of mixing for determining the parcel's level of neutral buoyancy strongly depends on the horizontal extent of the underlying heat source. Given a tropopause height ranging from 10-20 km, we expect that plumes with $R \sim 10$ km are relatively unaffected by entrainment during tropospheric ascent as the characteristic mixing length is 50 km. Plugging this mixing length into Eqn. 5.3, we find that $T_0^* \approx 66 \text{ K}$ is required to reach the stratosphere, close to the 60 K requirement for the adiabatic parcel. On the other hand, $R \sim 1$ km plumes possess $\ell \sim 5$ km and go through several e -folding dilutions before reaching the tropopause. In this case, Equation 5.3 gives $T_0^* \approx 300 \text{ K}$, five times larger than the adiabatic parcel.

The applicability of this result rests on several idealizations: 1) mixing can be described by a relaxation to environment of the form given in Eqn. 5.1, 2) the mixing length is accurately described by the MTT closure, and 3) the mixing length can be approximated by taking the plume's radius to be the source's radius. These assumptions can be tested and improved upon by examining the direct numerical simulation (DNS) of a turbulent plume presented in Tarshish and Romps (2021) (hereafter, TR2021). The DNS of TR2021 explicitly resolved the turbulent genesis of a plume by solving the non-dimensionalized Boussinesq equations above a no-slip circular source of buoyancy in an unstratified fluid.

Here we apply the mixing diagnosed in that unstratified simulation to our parcel rising through the stratified troposphere. This application is justified by prior findings that the MTT entrainment hypothesis applies equally well in stratified and unstratified environments: in both cases, experiments find $\alpha \sim 0.1$ (readers are directed to Kaye, 2008, for a review). In stratified flow at the plume's neutrally buoyant top, however, the entrainment model breaks down as the plume spreads horizontally, and the mixing length decouples from the plume's width. Despite this, the MTT model has been broadly successful at predicting LNB heights (Kaye, 2008).

The plume's Reynolds number in the DNS was 1000, well beneath that of a turbulent plume in Earth's atmosphere. However, as first noted by Scorer (1957), plume entrainment should asymptote to a universal limit that is independent of the Reynolds number once the flow is sufficiently turbulent. By progressively increasing the Reynolds number in two-

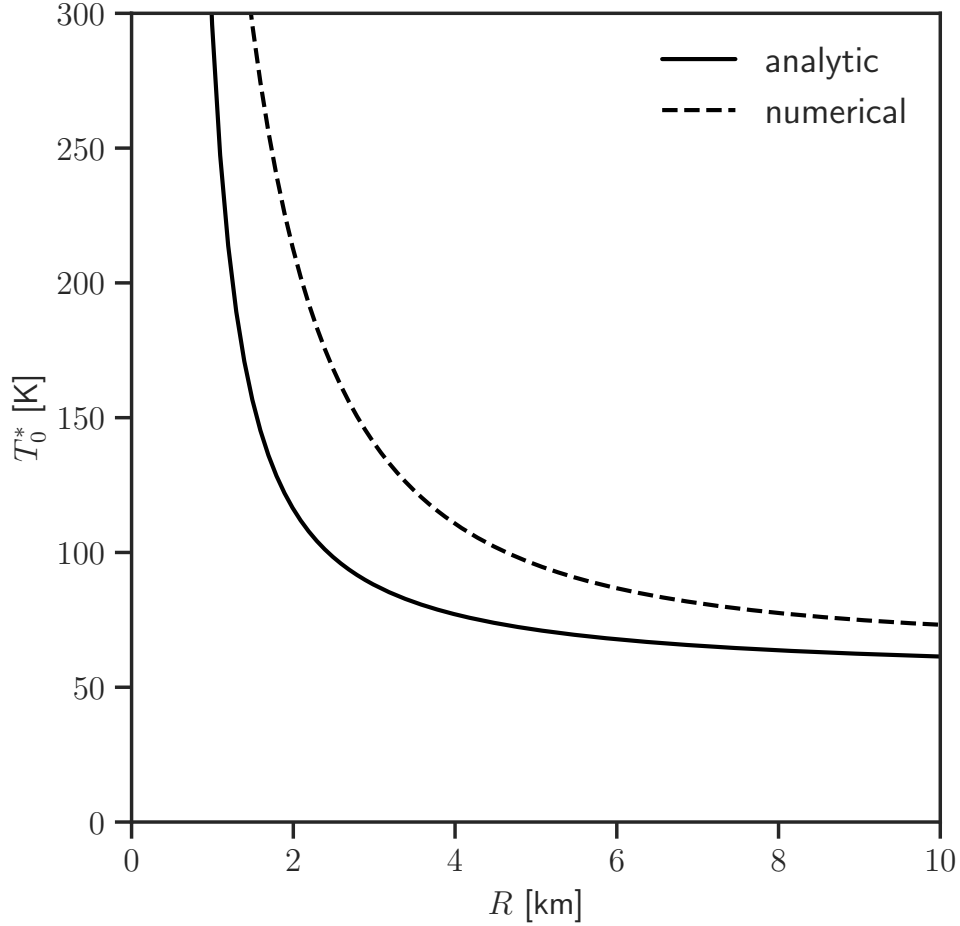


Figure 5.1: The temperature anomaly T_0^* required upon entering the free troposphere ($z_0 = 1$ km) for a dry, entraining parcel to remain buoyant up to the tropopause ($z_t = 15$ km) in an environment with lapse rate $\Gamma = 6 \text{ K km}^{-1}$. The analytic calculation (solid curve) shows Eqn. 5.3 evaluated with mixing closure $\ell = 5R$. The numerical solution (dashed curve) applies the mixing trajectory determined from a turbulent plume simulation; in this case, the parcel's dry static energy equation (Eqn. 5.1) is integrated with the empirical fit to the mixing length $\ell_{\text{DNS}}(z)$ shown in Figure 5.2.

dimensional simulations, TR2021 estimated that the three-dimensional results examined there were within 20% of this turbulent limit.

To apply the findings here, we first dimensionalize the DNS results by noting that a heat flux of $\mathcal{H}$ (units of kW/m²) sources buoyancy at a rate $F = g\mathcal{H}/(\rho c_p T_s)$ (units of m²/s³), where T_s is the environmental surface-air temperature. Given F and R , dimensional analysis yields the buoyancy profile $b(z) = f(z/R)F^{2/3}R^{-1/3}$, where f is a non-dimensional function that describes how the plume’s buoyancy decays with height. The mixing length is computed by differentiating the plume’s center-line buoyancy:

$$\ell_{\text{DNS}} = - \left(\frac{d \log b}{dz} \right)^{-1}. \quad (5.4)$$

The TR2021 results for $b(z)$ and ℓ_{DNS} are shown as the solid curves in the middle panels of Figure 5.2.

To enable future applications and reproducibility, we employ an empirical approximation for $b(z)$ that captures the behavior of the DNS near the surface and then asymptotes to the MTT solution. In terms of $\hat{z} = z/R$, the empirical fit is

$$f_{\text{fit}}(\hat{z}) = f_{\text{MTT}}(\hat{z}) - (2.06 + 14.7(1 - e^{-\hat{z}/0.018}))e^{-\hat{z}/0.56}, \quad (5.5)$$

where the MTT buoyancy scaling is

$$f_{\text{MTT}}(z) = b_0(\hat{z} - \hat{z}_v)^{-5/3}, \quad (5.6)$$

and TR2021 measured $b_0 = 31.3$ and $\hat{z}_v = -1.1$. While the DNS and MTT buoyancy curves converge with height, a slight offset persists in Figure 5.2 up to the sponge layer at $z \approx 7R$, where velocities and buoyancy are damped to achieve a steady state (readers are referred to section 3 of TR2021 for details). The empirical fit defaults to the MTT solution at large heights given the possibility that the DNS is affected by the overlying sponge.

As discussed in TR2021 and shown in Figure 5.2, the plume exhibits a near-surface layer in which mixing occurs over length-scales much smaller than the plume’s radius. This enhanced mixing is driven by a horizontal circulation that is unique to the surface. Above this layer, for $z > 2R$, the plume is well described by the MTT equations, and the plume’s radius converges to the analytical solution,

$$r(z) = \frac{6\alpha}{5} (z - z_v), \quad (5.7)$$

where z_v denotes the plume’s “virtual origin”, which TR2021 measured as $z_v \approx -1.1R$. In this MTT region, the diagnosed mixing length closely matches the $\ell = 5r$ parameterization, as shown in the second panel of Figure 5.2. The plume pinches up near the surface before entering the MTT regime with $r \approx R/3$, which sets the initial mixing length to roughly $\ell_{\text{DNS}} \approx 5R/3$. As the plume rises, it widens, eventually coming close to $r = R$ and $\ell_{\text{DNS}} \approx 5R$ at $z = 7R$.

We account for this height-dependent mixing by numerically integrating parcel ascent with mixing specified by an empirical fit to the DNS plume. This mixing profile is derived by substituting the empirical fit to the plume’s buoyancy (Eqn. 5.5) into the mixing length definition (Eqn. 5.4). Both height and the mixing length in the DNS are non-dimensionalized by the fire’s radius R . Therefore, specifying R determines a unique vertical profile of mixing between the parcel’s origin at $z_0 = 1$ km and the tropopause z_t . Given this mixing profile, we numerically integrate the parcel’s dry static energy equation (Eqn. 5.1) and solve for T_0^* , the parcel’s initial temperature that enables positively buoyant ascent up to z_t .

The environment is again specified by $z_t = 15$ km and a lapse rate of $\Gamma = 6$ K km⁻¹. This computation yields the dashed curve in Fig. 5.1. Comparing the numerical lifting result in Figure 5.1 to the analytic case, we observe higher initial temperature requirements, consistent with enhanced mixing resulting from the simulated plume’s radius being less than the fire’s radius.

While the quantitative details differ between the DNS and the analytic calculation, both indicate that if a dry plume enters the free troposphere with a temperature anomaly less than 60 K, then it will fall short of reaching the stratosphere. For plumes with radii less than 3 km, anomalies of hundreds of Kelvins are required to reach the tropopause in the presence of convective entrainment.

A more realistic environment

The above analysis assumed an idealized environment with a constant lapse rate of $\Gamma = 6$ K/km. Here we show that the findings hold more generally in the more realistic environment of ERA5 reanalysis (Hersbach et al., 2020). We compute the zonal-mean dry static energy over land and average over the time period 2010-2020. This yields the environmental profile $s_e(\phi, z)$, where ϕ is the latitude and z is height. To ease notation, let ϕ be implicit. We solve the entraining dry static energy equation (Eqn. 5.1) given the plume’s initial $s(z_0) = c_p T'_0 + s_e(z_0)$, yielding

$$s(z) = e^{-(z-z_0)/\ell} s(z_0) + \frac{e^{-z/\ell}}{\ell} \int_{z_0}^z e^{z'/\ell} s_e(z') dz', \quad (5.8)$$

where for simplicity we have assumed that ℓ is constant. Define $T_0^*(z)$ as the *initial temperature anomaly* at the base of the free troposphere (z_0) required of a dry parcel for the parcel to be positively buoyant up to z . This can be found by noting that the level of neutral buoyancy occurs where $s = s_e$. Applying this constraint, and solving for $T_0^*(z)$ results in

$$T_0^*(z) = c_p^{-1} \left(s_e(z) e^{(z-z_0)/\ell} - s_e(z_0) - \ell^{-1} e^{z_0/\ell} \int_{z_0}^z e^{z'/\ell} s_e(z') dz' \right). \quad (5.9)$$

Applying this formula to the ERA5 data and setting $z_0 = 1$ km yields Figure 5.3. We illustrate the tropopause layer with the top and bottom contours given by the cold point

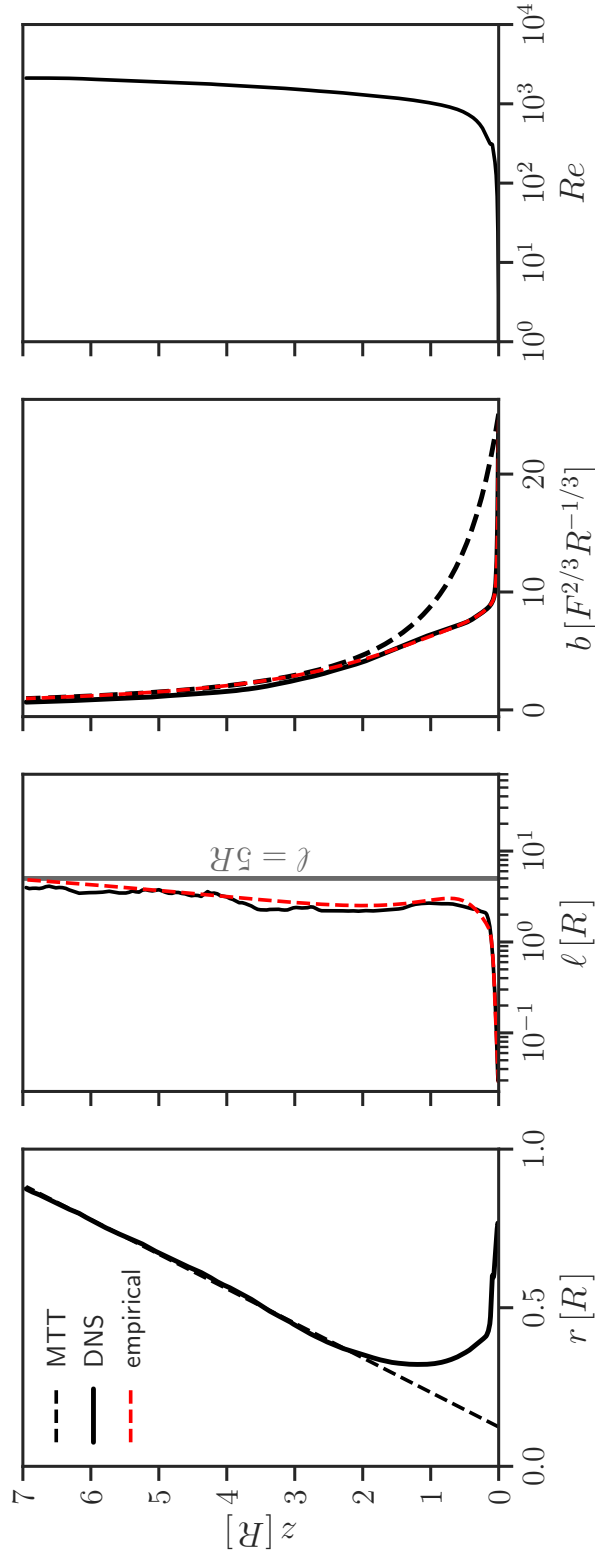


Figure 5.2: Radius, mixing length, buoyancy, and Reynolds number $Re = wr/\nu$ diagnosed in the direct numerical simulation of TR2021 (solid black), along with the MTT scalings (dashed black), the analytic $\ell = 5R$ closure (gray), and the empirical fits to the buoyancy and mixing length (red dashed).

definition and World Meteorological Organization (WMO) lapse-rate definition, respectively. The WMO tropopause is the lowest level where the absolute value of the temperature lapse rate decreases to 2K/km, or less, with the average lapse rate between this level and all higher levels within 2 km not exceeding 2K/km (Hoffmann and Spang, 2021). The “dynamic” tropopause, a potential vorticity contour of +3.5 PVU, is also shown. The zonal and time average tropopause contours are computed from the ERA5 tropopause dataset (Hoffmann and Spang, 2021).

Consistent with the idealized calculation, we find that reaching the tropical tropopause requires a temperature anomaly of roughly 60 K for an adiabatic parcel. For weak entrainment, corresponding to the plumes with a fire radius of 10 km (thus $l = 50$ km), the entraining and adiabatic results are nearly identical in the troposphere. Decreasing the fire radius to 1 km (implying $l = 5$ km) strands the +60 K plume in the middle of the tropical troposphere; the initial temperature anomaly must be increased to ~ 300 K to reach the tropical stratosphere. At high latitudes, the lower tropopause entails fewer e -folding dilutions for a parcel ascending to the stratosphere, and the temperature requirement is less sensitive to the degree of mixing. A polar adiabatic parcel requires ~ 40 K for stratospheric ascent, and strong mixing ($\ell = 5$ km) roughly doubles this requirement. Given the focus on nuclear winter, the geographic areas of importance here and in prior studies (Wagman et al., 2020; Jägermeyr et al., 2020; Reisner et al., 2018; Mills et al., 2014) are the mid-latitudes and tropics. Not only do these regions contain the majority of urban areas, but stratospheric dynamics raise the risk of nuclear winter; emplacing material in the ascending branch of the stratospheric overturning circulation maximizes the residence time and areal extent (Butchart, 2014).

5.3 What T'_0 can urban firestorms generate?

Now that we have estimates of the temperature required for dry plumes to reach the stratosphere, we investigate whether firestorms do in fact produce plumes that are hot enough to satisfy these requirements.

DNS and LES setups

While MTT provides an analytic theory for the far-field plume, there is no analytic theory for the plume near the surface. To provide estimates of the plume’s temperature (T'_0) in the region of interest, we will rely here on simulations. We will use the DNS of TR2021 as well as large-eddy simulations (LES) that approximate rather than resolve turbulent dissipation. Each paradigm has its weaknesses (inability to simulate a high Reynolds number with DNS, and limited accuracy of the turbulence closure for LES) so comparing and combining them enables a more robust estimate.

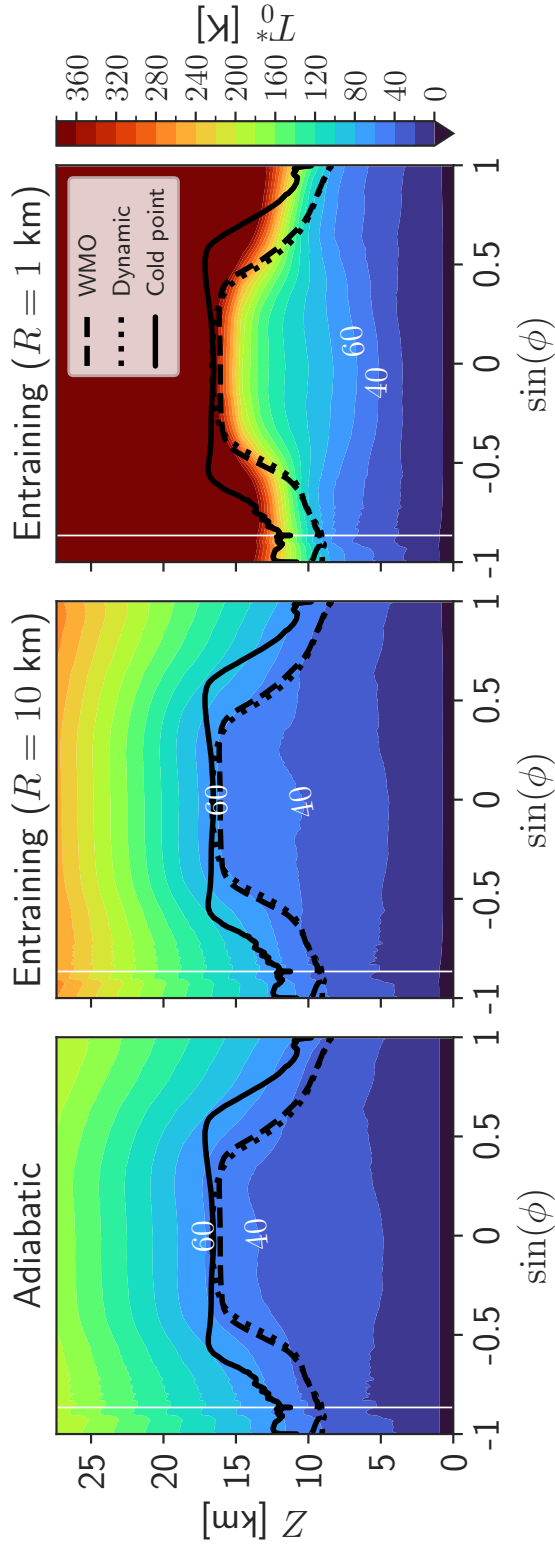


Figure 5.3: Temperature anomaly $T_0^*(z)$ required of a dry parcel at $z_0 = 1$ km to ascend to a given height under adiabatic motion (left) or entrainment with the $\ell = 5R$ closure (center and right) versus $\sin$ of the latitude ϕ . A $R = 10$ km plume (center) has $\ell = 50$ km and thus resembles the adiabatic result in the troposphere. Stronger mixing, with $\ell = 5$ km corresponding to a plume with $R = 1$ km (right), raises the required temperature anomaly by multiples ranging from a factor of two (poles) to six (tropics). The tropopause is shown as defined by the cold point, dynamic definition, and the WMO lapse-rate definition (Hoffmann and Spang, 2021). Vertical axis is log-pressure $Z = -H \log(p/p_r)$ with scale height $H = 8$ km and reference pressure $p_r = 1000$ mb.

We evaluate the empirical fit to the DNS $b(z)$ plotted in Figure 5.2 at $z = z_0 = 1$ km to find the initial buoyancy $b(z_0)$ and the temperature anomaly $T'_0 = T_e(z_0)b(z_0)/g$ for a range of fire radii and heat fluxes. For $R = 1 - 10$ km and $z_0 = 1$ km, the initial non-dimensional heights are $\hat{z}_0 = z_0/R = 0.1 - 1.0$ in the DNS. Over this height range, the local Reynolds number of the plume increases from roughly 300 to 1000 as shown in the right panel of Figure 5.2. Therefore, a $R = 10$ km plume, which has $\hat{z}_0 = 0.1$, is not yet fully turbulent at $z = z_0$ in the simulations. This is a limitation of the model, since plumes in Earth's atmosphere are fully turbulent at $z_0 = 1$ km. Increasing the global Reynolds number from 500 to 10^4 in two-dimensional simulations changes the buoyancy at $\hat{z}_0 = 0.1$ by less than 15% (TR2021, Figure 2). This weak sensitivity to Reynolds number suggests that the three-dimensional DNS may provide a reliable estimate for the atmospheric plume's buoyancy over the height range analyzed here, despite the simulation's semi-turbulent nature close to the surface.

We compare the DNS to LES performed with the fully-compressible, cloud-resolving Das Atmosphärische Modell (DAM) detailed in Romps (2008). Simulations are conducted with an unstratified fluid and a surface temperature of 300 K. One set of simulations has the source radius R varying over [500 m, 1 km, 2 km, 4 km, 7 km] and $\mathcal{H} = 50$ kW m⁻². An additional set is run with $\mathcal{H} = [10, 50, 100]$ kW m⁻² and $R = 5$ km. The setup is configured to match the DNS run described in the appendix of TR2021. Each simulation has a cubic domain with side length $L = 4R$. The flow is damped with Rayleigh drag and Newtonian cooling outside of a half ellipse centered at the surface. (Given the dry lapse rate and surface temperature, the maximum domain height with temperatures above absolute zero is 30 km; given $L = 4R$, this is why the largest R tested is 7 km and not 8 km.)

For each simulation, the chosen heat flux $\mathcal{H}$ specifies the buoyancy flux F , which, along with the fire radius R , gives the characteristic timescale $T = F^{-1/3}R^{2/3}$. Simulations are run for $35T$, steady state is achieved after roughly $10T$, and averages are taken over the last $25T$. The resolution is uniform with $N = 256$ grid cells in each direction. To test for resolution sensitivity, we branch off pairs of additional simulations from the end of the $R = 1$ km and $R = 7$ km simulations. These tests are run for an additional $10T$ with lower resolution ($N = 128$) and higher resolution ($N = 512$), respectively.

Results

The left panel of Figure 5.4 shows the results for the DNS and LES cases given varying fire radii. Both the LES and DNS exhibit a global maximum in T'_0 (the plume's temperature at $z = 1$ km) for $R \approx 1$ km. Two competing effects result in this maximum at intermediate R . Near the surface, a horizontal circulation of characteristic velocity $U \sim F^{1/3}R^{1/3}$ ventilates the starting layer of buoyancy $B \sim F^{2/3}R^{-1/3}$, a decreasing function of R . However, the buoyancy dilution as the plume rises is set by the mixing length, which is an increasing function of R . The trade-off between maximizing the starting buoyancy (which favors a smaller plume) and preserving the buoyancy during ascent (which favors a larger plume) yields the non-monotonic dependence of T'_0 on R . The left panel of Figure 5.4 shows a

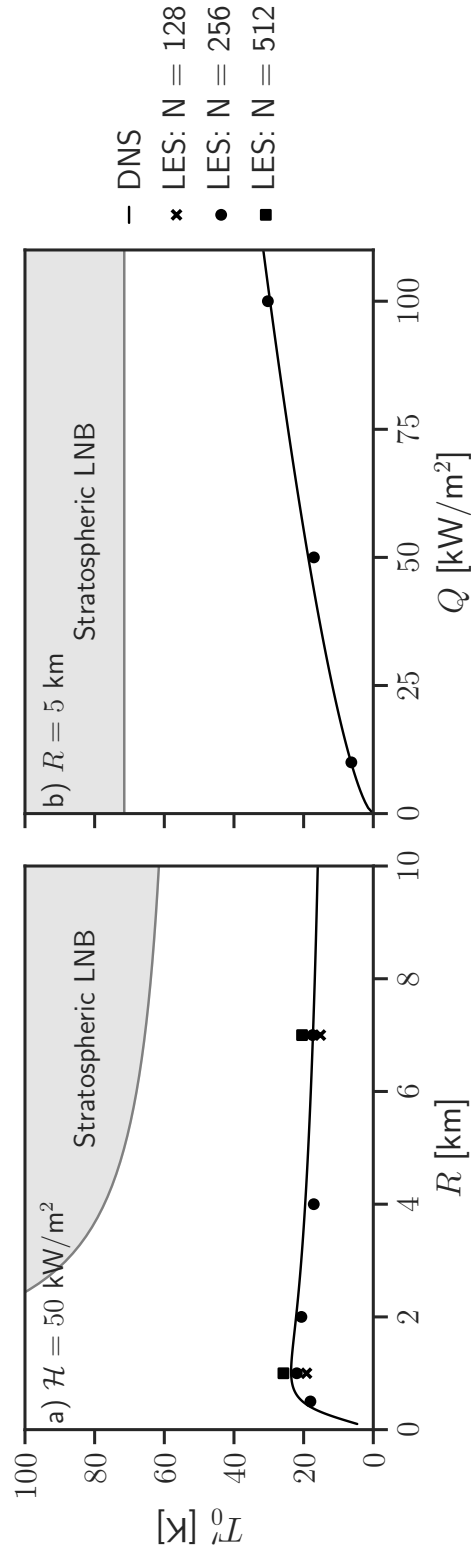


Figure 5.4: Comparison of the temperature anomaly at 1 km (T'_0) for dry plumes in an unstratified fluid given sources with varying radii R (a) and heat fluxes $\mathcal{H}$ (b) in the direct numerical simulation (solid curve) and the large-eddy simulations (scatter). For the $R = 1$ km and 7 km cases, simulations at different resolutions are labeled by the number of grid cells N along each dimension in the uniform grid. The gray area indicates the temperatures that enable stratospheric ascent using, as a conservative estimate, the analytic case in Figure 5.1.

maximum magnitude T'_0 of approximately 25 K at $R \approx 1$ km. We estimate the uncertainty in T'_0 to be roughly 10 K, the maximum spread between the DNS simulation and the LES simulations, and also roughly the sensitivity of the LES simulations to two doublings in resolution.

Turning to the varying heat flux simulations in the right panel of Figure 5.4, we observe that the LES estimate is broadly consistent with the DNS scaling $T'_0 \propto \mathcal{H}^{2/3}$. Using this scaling, the $T'_0 \approx 20$ K result for a $R = 10$ km and $\mathcal{H} = 50 \text{ kW m}^{-2}$ plume in Figure 5.4 implies that increasing this fire's $\mathcal{H}$ to 100 kW m^{-2} yields a plume with $T'_0 \approx 2^{2/3}20 \approx 32$ K. This plume has the largest radius and thus the lowest temperature requirement for stratospheric ascent of any plume in Figure 5.4. Combined with a heat flux at the upper end of the plausible range for firestorms, this plume has the most favorable properties for stratospheric ascent. Despite these optimal conditions, the plume's temperature anomaly is less than the stratospheric requirement by roughly a factor of two.

Based on this finding, we conclude that firestorm plumes cannot reach the tropical stratosphere under dry ascent. This conclusion points to a clear role for latent heating in lofting soot to the stratosphere. After all, latent heating powers ordinary deep convection to the tropopause despite much less intense boundary-layer sources. The tropical atmosphere is approximately neutrally stable to an ordinary moist entraining plume, with an entrainment rate that is comparable to firestorm plumes and a T'_0 that is smaller by one or two orders of magnitude. It therefore stands to reason that a moist firestorm plume would easily ascend to the tropopause in places and times that are already primed to host deep convective storms.

We have applied unstratified simulations to estimate the plume's temperature anomaly at the top of the atmospheric boundary layer. This approach neglected the potential effects of the overlying troposphere on the plume's initial development. The accuracy of this approximation can be estimated by appealing to the characteristic length-scale over which stratification influences the plume,

$$\frac{b}{N^2} = \frac{T'_0}{g/c_p - \Gamma} \sim \frac{30\text{K}}{4\text{K/km}} = 7.5 \text{ km}, \quad (5.10)$$

where N is the Brunt-Väisälä frequency. Given that b/N^2 is substantially larger than z_0 , we expect that the overlying stratification has a limited impact on the boundary-layer ascent. To confirm, we ran an alternate LES simulation (not shown) with the environment set equal to the U.S. standard atmosphere (Minzner, 1977) with a $R = 5$ km and $\mathcal{H} = 50 \text{ kW m}^{-2}$ plume. The difference in T'_0 between the stratified and unstratified case is roughly 5 K, suggesting that the unstratified simulations provide valid first-order approximations for our purposes.

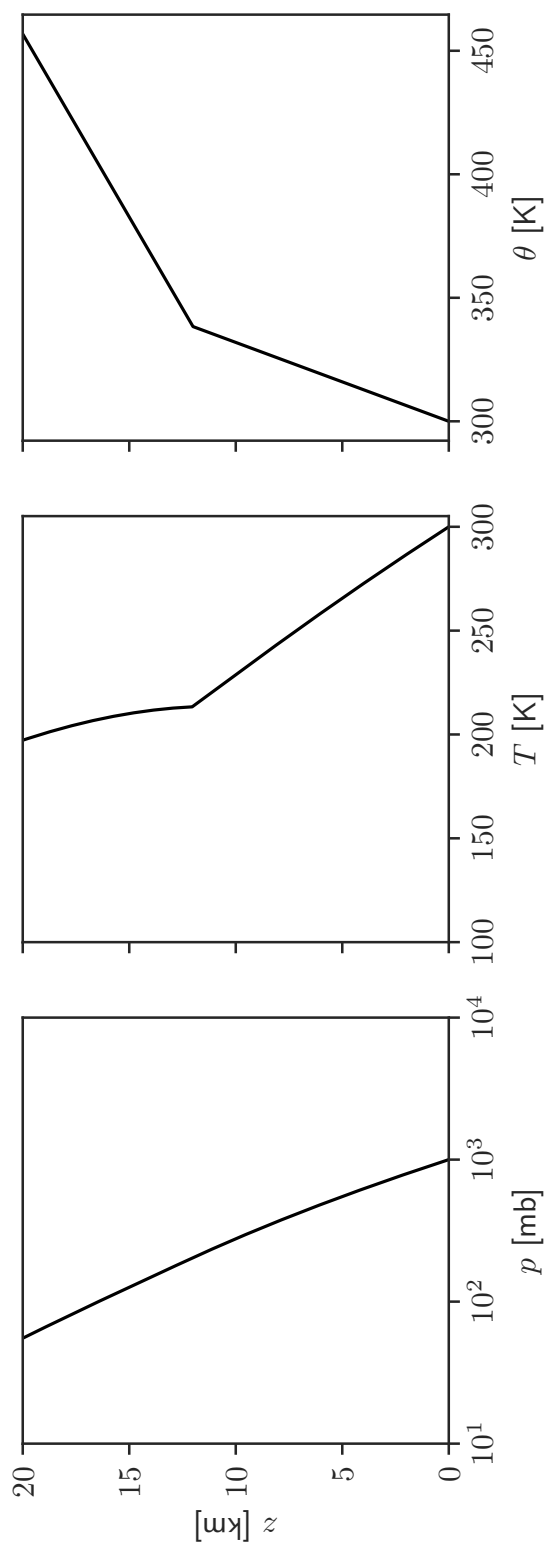


Figure 5.5: Environmental pressure (left), temperature (center), and potential temperature (right) in the cloud-resolving simulations corresponding to the Reisner et al. (2018) case study.

5.4 Case study

The recent study of Reisner et al. (2018) performed dry simulations of a firestorm plume and found that the plume became neutrally buoyant in the lower atmosphere, consistent with the findings here. Given access to moisture, however, we expect that firestorm plumes behave like ordinary deep convective events, which rely on latent heating to reach the upper atmosphere.

We illustrate the importance of latent heating by performing a moist version of the Reisner et al. (2018) simulation. The plume in Reisner et al. (2018) originates from a nuclear blast simulation and fire model, but exact surface fluxes were not reported or archived (personal communication). We estimate from Figure 2 of Reisner et al. (2018) that the actively burning core of the near circular firestorm has $R \approx 2$ km. The average burnable fuel load inside this area is estimated to be 9.1 kg m^{-2} (Robock et al., 2019), which, when combined with an approximate e -folding burn time of 1 hour and the $1.5 \times 10^7 \text{ J kg}^{-1}$ energy content of dry wood (Toon et al., 2007), yields a characteristic heat flux of

$$\mathcal{H} = \frac{(9.1 \text{ kg m}^{-2}) (1.5 \times 10^7 \text{ J kg}^{-1})}{3600 \text{ s}} \approx 40 \text{ kW m}^{-2}. \quad (5.11)$$

This magnitude of heat flux is comparable to the other firestorm cases in Table 1. The environment is taken to be stably stratified and specified by a piece-wise linear potential temperature profile. The lapse rates are estimated from Figure 3 of Reisner et al. (2018) to be $d\theta/dz = 3.2 \text{ K km}^{-1}$ beneath the 12 km tropopause and $d\theta/dz = 14.8 \text{ K km}^{-1}$ above. Figure 5 shows the environmental profile, derived by applying hydrostatic balance above a surface with temperature 300 K and pressure 1000 mb. The temperature lapse rate is near constant in the troposphere and averages to 7.2 K km^{-1} .

The large-eddy simulations presented here are run with a circular fire of radius $R = 2$ km and heat flux $\mathcal{H} = 40 \text{ kW m}^2$. The heat flux is constant over the hour-long simulation. The environment is initially at rest and specified by the Reisner et al. (2018) setup except for the addition of moisture. We run a dry simulation and a moist simulation with a constant vertical profile of relative humidity of 70%. The DAM domain has horizontal dimensions of $48 \text{ km} \times 48 \text{ km}$ and a height of 32 km. The horizontal grid spacing is $\Delta x = \Delta y = 150 \text{ m}$ and the vertical grid spacing increases smoothly from $\Delta z = 10 \text{ m}$ at the surface to $\Delta z = 50 \text{ m}$ at 1 km. For $z > 1 \text{ km}$, Δz again increases to 100 m at 10 km, and then to 300 m at the domain top. To identify the plume’s level of neutral buoyancy, we advect “soot,” taken here to be a tracer fixed to 1 throughout the simulation in the bottom grid cells that overlie the surface fire, and initialized as 0 elsewhere. We do not model combustion or actual soot emissions. The soot tracer also has no impact on the moist simulation’s microphysics, which is modeled with a six-class Lin-Lord-Krueger scheme (Lin et al., 1983; Lord et al., 1984; Krueger et al., 1995). Vigorous convection develops within the first few minutes of the simulations, and the vertical soot flux approaches a steady state after roughly fifteen minutes. Soot then begins to accumulate and spread horizontally at the LNB. The large horizontal extent of the domain

ensures that the spreading plume top is at least 5 km away from the periodic boundaries at all times.

Figure 5.6(a) plots a cross section of the soot field at the end of the dry simulation and demonstrates that the dry plume does not reach the upper atmosphere. Soot accumulates instead at ~ 5 km. Our results are consistent with Reisner et al. (2018), who found that their dry plume became neutrally buoyant in the lower atmosphere. Figure 6 in Reisner et al. (2018), in particular, shows a dry LNB of ~ 3 km. The moist simulation, however, rises to the tropopause at 12 km, confirming our expectation: firestorm plumes are able to reach the stratosphere by tapping into the abundant latent heat available in a humid atmosphere.

The surface humidity required to power stratospheric ascent is expected to depend on the environmental stratification. For the particular stratification analyzed here, we explore the sensitivity of the LNB to the abundance of moisture by increasing the relative humidity from 0 to 0.9 by increments of 0.1 in the Reisner et al. (2018) setup. Figure 5.7 demonstrates how the vertical soot profile depends on the relative humidity. For $RH > .5$, the soot’s LNB is at or above the tropopause height of 12 km. As RH decreases, the LNB lowers from the upper troposphere ($RH \sim .5$) to the middle troposphere ($RH \sim .4$) and then demonstrates a secondary lower maxima ($RH = .3$). Only for $RH \leq .2$ does the moist simulation closely resemble the dry simulation. These results suggest that moisture is essential to firestorm plume rise even in relatively dry environments.

While we do not simulate different stratifications, boundary-layer depths, wind shears, relative humidity profiles, or tropopause altitudes, we expect that our central finding—latent heating is required for stratospheric firestorm ascent—will generally hold in these other environments. This expectation is in part based on the thermodynamic parcel calculations of section 2 and 3. The temperature requirements for dry stratospheric ascent were found to be fairly uniform across latitudes (Figure 5.3), suggesting only a weak sensitivity to variations in stratification, boundary-layer depth, and tropopause height. Wind shear is not incorporated into this parcel calculation, but would be expected to increase plume entrainment, and thus further inhibit dry plume ascent. Additional support for the broad applicability of our findings comes from simulations of other firestorms, which showed that dry plumes were unable to reach the stratosphere across a wide array of environmental conditions (Wagman et al., 2020; Redfern et al., 2021).

5.5 Discussion

Firestorm plumes lift sooty surface air up into the atmosphere. The height at which this soot accumulates determines if the soot is scavenged by precipitation from the lower atmosphere over a few days, or if the soot resides above the weather in the stratosphere for months to years. Soot in the stratosphere dims sunlight at the surface, and megatons of stratospheric soot are thought to yield catastrophic global cooling. The role of latent heating in enabling this stratospheric ascent is a subject of recent debate.

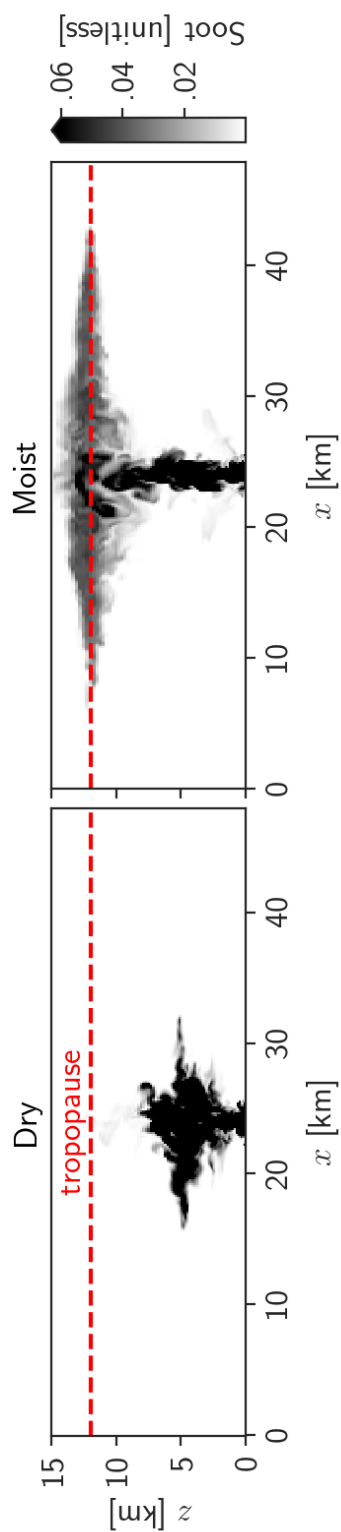


Figure 5.6: Comparison of dry (left) and moist (right) simulations. Soot cross sections are evaluated at the end of simulations and in the center of the domain. The moist simulation has 70% relative humidity.

Here, we show that dry ascent is not sufficient to lift firestorm soot to the tropical and mid-latitude stratosphere. We probe dry plume ascent by studying the plume’s temperature anomaly T'_0 as it enters the free troposphere. Simple lifting calculations indicate that a dry adiabatic plume requires a T'_0 of roughly 60 K to reach the tropical tropopause. The accuracy of this adiabatic calculation is shown to depend on the size of the plume’s heat source. The plume’s radius determines the mixing length ℓ , and mixing becomes a first-order effect if ℓ is comparable to the tropopause height. For firestorm plume’s with narrow sources ($R \sim 1$ km), we find that ℓ is a few kilometers. As a result, environmental mixing has a large impact on the ascent, and raises the required temperature anomaly to hundreds of Kelvin.

Both DNS and LES simulations indicate that firestorm plumes with heat sources in the range $\mathcal{H} = 10 - 100 \text{ kW m}^{-2}$ yield plumes with T'_0 at $z_0 = 1$ km in the low tens of Kelvin. We show that this is largely independent of R , and somewhat counter-intuitively, we demonstrate that increasing R beyond a certain value reduces T'_0 . A smaller fire radius increases the surface buoyancy, but a larger fire radius lowers the entrainment rate as the plume rises through the boundary layer. An intermediate fire radius has the maximal T'_0 due to these competing effects. For subsequent ascent beyond the boundary layer, a larger radius and correspondingly lower entrainment rate is again advantageous for the plume. This entrainment effect dominates such that a larger R is always more advantageous for reaching the stratosphere. Taken together, the simulations indicate that firestorm plumes fall short of the temperature requirements to reach the tropical stratosphere under dry ascent by a factor of at least two.

This claim is compatible with the case study presented here and the prior simulations of Wagman et al. (2020), Reisner et al. (2018), and Penner et al. (1986), who all found dry plume LNBs beneath the stratosphere. The balance of evidence indicates that the stratospheric dry plume of Small and Heikes (1988) is an outlier. We speculated that the two-dimensional nature of the simulations in Small and Heikes (1988) might be responsible for the discrepancy. To investigate, we ran two-dimensional cases in DAM (not shown), but did not find that the dimensionality had a significant effect on the dry LNB. Therefore, an explanation of the stratospheric dry LNB in Small and Heikes (1988) remains outstanding.

With the addition of moisture, however, we find that firestorm plumes are indeed able to reach the stratosphere. This finding advances the view that firestorm plumes, like ordinary deep convection, reach lofty heights by tapping into the abundant latent heat present in a moist environment. In this view, moisture is a key determinant of plume height, and its essential role was overlooked in prior ascent models (Manins, 1985; Carrier et al., 1985). Our findings indicate that dry simulations should not be used to investigate firestorm plume lofting and cast doubt on the applicability of past research (Reisner et al., 2018, e.g.,) that neglected latent heating.

A limitation of the theory and simulations presented here is the absence of soot microphysics. Soot aerosols provide cloud condensation nuclei that may alter the drop size distribution and impact auto-conversion. This aerosol effect is expected to invigorate convection (Lee et al., 2020), lofting the plume higher. Coupling soot to microphysics, however,

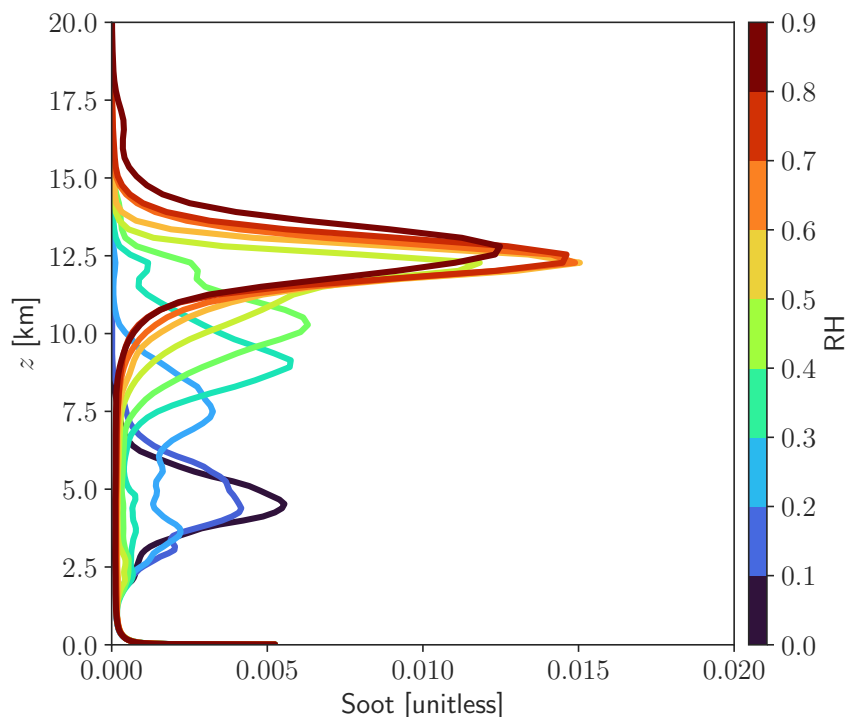


Figure 5.7: Final vertical profile of soot for simulations with relative humidity (color) varying from 0% to 90% by increments of 10%.

also enables soot to rain out, which could remove much of the soot from the rising plume as suggested in Penner et al. (1986). Given the essential role of moisture in lofting firestorm plumes we identified here, future research should investigate how these second-order microphysical effects impact firestorm soot transport. Another aspect not addressed here and deserving of future study is the radiative lofting of plumes, which has been observed to substantially lift wildfire plume soot for months after the fire (Yu et al., 2019).

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